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Dr. J. H. JEANS, PRESIDENT, in the Chair.

Dr. E. S. Beaven, Eastway, Warminster;
Thomas MacFarland Cherry, Ph.D., Fellow of Trinity College, Cambridge;
Clarendon Ions, 35 Southeast First Avenue, Miami, Florida, U.S.A.;
Takehiko Matukuma, Assistant Professor of Astronomy, Tôhoku Imperial University, Sendai, Japan;
Maurice Rupert Meehan, 6 Bridge Street, Evesham, Worcestershire;
Mrs. Dorothea Klumpke-Roberts (Mrs. Isaac Roberts), Chateau de By-par-Thomery, Seine et Marne, France;
Clifford Somes Saxton, M.A., Whatley House, near Frome, Somerset; and
Francis Joseph Venning, Milton House, Arnison Road, East Molesey, Surrey,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Ethel Frances Batwell Bellamy, Assistant at the University Observatory, Oxford, 28 Polstead Road, Oxford (proposed by H. C. Plummer);

George Frederick Morrell, Avenue House, Holly Park, Crouch Hill, London, N. 4 (proposed by T. H. L. Hony); and
Winthrop Robins Wright, Professor of Physics, Swarthmore College, Swarthmore, Pa., U.S.A., and Cavendish Laboratory, Cambridge (proposed by L. J. Comrie).

Seventy-six presents were announced as having been received since the last meeting, including, amongst others :—

Tycho Brahe, Opera Omnia, vol. 12 (presented by Dr. Dreyer).
Benson, D. E., The Making of a Speculum (presented by the Liverpool Astronomical Society).
Astrographic Catalogue: Algiers Observatory, vols. 1, 2, 3; Paris Observatory, vol. 5; Rome, Specola Vaticana, vol. 8; Toulouse Observatory, vol. 7.
Astrographic Chart: Bordeaux Observatory, zones $+12°$, $+14°$, $+16°$ (12 charts); Paris Observatory, zones $+18°$, $+20°$ (5 charts); Toulouse Observatory, zones $+5°$, $+7°$, $+9°$ (9 charts).
Albany Zone Catalogue for the epoch 1900; prepared at the Dudley Observatory, Albany, N.Y. (presented by the Carnegie Institution of Washington).

The Radiation from a Pulsating Star and from a Star in Process of Fission. By J. H. Jeans.

1. The radiation from a Cepheid variable conforms fairly well with what might be expected from a pulsating star except for two somewhat surprising features :—

(1) The intensity of radiation appears to be in phase with $\dot{r}$, the rate at which the radius of the star is increasing, rather than with r, the length of the radius. For most Cepheids maximum luminosity coincides fairly closely with maximum velocity of approach.

(2) The light-curves, whether examined individually or statistically, show no fore-and-aft symmetry, as though the motion were not reversible in time.

Probably any theory of Cepheid variation will stand or fall according to its ability to explain these two phenomena. The present paper examines to what extent the spherical pulsating star of Shapley and Eddington conforms to these requirements, and also contains a preliminary examination of the position with respect to the theory of Cepheid variation I suggested in *M.N.*, 1925 June, according to which a Cepheid is a star in process of breaking up into two by rotational fission.

2. We may conveniently start from the equations of a pulsating sphere. These are (*M.N. Suppl.*, 1925, pp. 916, 917)

$$\frac{\delta\rho}{\rho}+\frac{2\delta r}{r}+\frac{d}{dr}(\delta r)=0 \quad . \quad . \quad . \quad . \quad (1)$$

$$\frac{\delta H}{H}=\frac{(6+n)\delta T}{T}-\frac{2\delta\rho}{\rho}-\frac{d}{dr}(\delta r)+\frac{d\delta T}{dr}\Big/\frac{\partial T}{\partial r} \quad . \quad . \quad (2)$$

$$-\frac{d}{dr}\delta(r^2H)=r^2\left[\rho C_v\frac{d}{dt}(\delta T)-P\frac{d}{dt}\left(\frac{\delta\rho}{\rho}\right)\right] \quad . \quad . \quad (3)$$

$$\frac{d^2}{dt^2}(\delta r)=\left[\frac{4\pi M_r}{r^3}+\left(\frac{2p}{\rho r}+\frac{1}{\rho}\frac{dp}{dr}\right)\frac{d}{dr}+\frac{p}{\rho}\frac{d^2}{dr^2}\right]\delta r-\frac{1}{\rho}\frac{d}{dr}\left[(p+\tfrac{4}{3}aT^4)\frac{\delta T}{T}\right]$$
$$+\frac{4\mu}{3}\frac{\partial}{\partial r}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{d}{dt}(\delta r)\right)\right] \quad . \quad (4)$$

In these equations each element initially at a distance r from the centre is supposed to move to a distance $r+\delta r$, ρ and T changing to $\rho+\delta\rho$ and $T+\delta T$; C_v stands for the total specific heat at constant volume of matter, including ionisation, and radiation; P is the total pressure $p+\frac{1}{3}aT^4$; the last term in equation (4), which was not included in my earlier paper, represents the effect of viscosity,* μ being the usual kinetic coefficient of viscosity.†

3. As was shown in the paper already quoted, the general solution has a time-factor $e^{-\lambda t}\cos(qt-\epsilon)$. In virtue of equation (1) we may assume that

$$r=r_0+r_1e^{-\lambda t}\cos qt \quad . \quad . \quad . \quad . \quad (5)$$

$$\rho=\rho_0+\rho_1e^{-\lambda t}\cos qt \quad . \quad . \quad . \quad . \quad (6)$$

where r_1 and ρ_1 are functions of r connected by the relation

$$\frac{\rho_1}{\rho_0}+\frac{2r_1}{r_0}+\frac{dr_1}{dr_0}=0 \quad . \quad . \quad . \quad . \quad (7)$$

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* *Cf.* Lamb's *Hydrodynamics*, 5th ed., p. 546.

† *Added in proof.* On the other hand, equation (3) originally contained a term $+\rho r^2\delta G$ in its left-hand member, this representing the influence of changes in the rate of generation of energy per unit volume G. This term was omitted in the present paper because I had shown in the earlier paper that δG must be zero for stars of the spectral type and luminosity of Cepheids. When the present paper was read it was suggested in discussion that the retention of the term in δG might have provided an escape from the uniformity of phase of r, ρ, T, and H. This does not, however, appear to be the case. If G is a function of T and ρ, δG will be in phase with δT and $\delta\rho$, and therefore with the term $-\frac{d}{dr}\delta(r^2H)$ on the left of equation (3). The whole argument is as before, except that the numerator of formula (16) must be replaced by

$$\frac{d}{dr}(r^2H)-\rho^2r^2\frac{\partial\delta G}{\partial\rho},$$

after which the conclusions of the paper follow as before.

We now have

$$\frac{d}{dt}(\delta r)=r_1e^{-\lambda t}(-\lambda\cos qt-q\sin qt) \quad . \quad . \quad . \quad (8)$$

$$\frac{d^2}{dt^2}(\delta r)=r_1e^{-\lambda t}((\lambda^2-q^2)\cos qt+2\lambda q\sin qt) \quad . \quad . \quad (9)$$

Thus the solution of equation (4) must be of the form

$$\mathrm{T}=\mathrm{T}_0+\mathrm{T}_1e^{-\lambda t}\cos qt+\lambda\mathrm{T}_2e^{-\lambda t}\sin qt \quad . \quad . \quad (10)$$

Inserting this value into equation (4) and equating coefficients of $\cos qt$ and $\sin qt$, we obtain

$$r_1(\lambda^2-q^2)=\left[\frac{4\pi\mathrm{M}_r}{r^3}+\left(\frac{2p}{\rho r}+\frac{1}{\rho}\frac{dp}{dr}\right)\frac{d}{dr}+\frac{p}{\rho}\frac{d^2}{dr^2}\right]r_1-\frac{1}{\rho}\frac{d}{dr}\left[(p+\tfrac{4}{3}a\mathrm{T}^4)\frac{\mathrm{T}_1}{\mathrm{T}}\right]$$
$$-\frac{4\mu}{3}\lambda\frac{d}{dr}\left[\frac{1}{r^2}\frac{d}{dr}(r^2r_1)\right] \quad . \quad (11)$$

$$2\lambda qr_1=-\frac{\lambda}{\rho}\frac{d}{dr}\left[(p+\tfrac{4}{3}a\mathrm{T}^4)\frac{\mathrm{T}_2}{\mathrm{T}}\right]-\frac{4\mu}{3}q\frac{d}{dr}\left[\frac{1}{r^2}\frac{d}{dr}(r^2r_1)\right] \quad . \quad (12)$$

With the assumed values for r, ρ, and P equation (3) takes the form

$$-\frac{d}{dr}\delta(r^2\mathrm{H})=r^2e^{-\lambda t}\rho\mathrm{C}_v[\mathrm{T}_1(-\lambda\cos qt-q\sin qt)+\lambda\mathrm{T}_2(-\lambda\sin qt+q\cos qt)]$$
$$-r^2e^{-\lambda t}\frac{\mathrm{P}\rho_1}{\rho}(-\lambda\cos qt-q\sin qt) \quad . \quad (13)$$

while equation (2) gives

$$\delta(r^2\mathrm{H})=r^2\mathrm{H}\left[(6+n)\frac{\mathrm{T}_1}{\mathrm{T}}+\frac{2r_1}{r}-\frac{2\rho_1}{\rho}-\frac{dr_1}{dr}+\frac{d\mathrm{T}_1}{dr}\Big/\frac{d\mathrm{T}}{dr}\right]e^{-\lambda t}\cos qt$$
$$+r^2\mathrm{H}\lambda\left[(6+n)\frac{\mathrm{T}_2}{\mathrm{T}}+\frac{d\mathrm{T}_2}{dr}\Big/\frac{d\mathrm{T}}{dr}\right]e^{-\lambda t}\sin qt.$$

Eliminating $\delta(r^2\mathrm{H})$ between this equation and (13) and equating coefficients of $\cos qt$ and $\sin qt$ respectively, we find

$$r^2\rho\mathrm{C}_v[-\lambda\mathrm{T}_1+q\lambda\mathrm{T}_2]+r^2\lambda\mathrm{P}\frac{\rho_1}{\rho}$$
$$=-\frac{d}{dr}\left\{r^2\mathrm{H}\left[(6+n)\frac{\mathrm{T}_1}{\mathrm{T}}+\frac{2r_1}{r}-\frac{2\rho_1}{\rho}-\frac{dr_1}{dr}+\frac{d\mathrm{T}_1}{dr}\Big/\frac{d\mathrm{T}}{dr}\right]\right\} \quad . \quad (14)$$

$$r^2\rho\mathrm{C}_v\left[-q\mathrm{T}_1-\lambda^2\mathrm{T}_2\right]+r^2q\mathrm{P}\frac{\rho_1}{\rho}=-\lambda\frac{d}{dr}\left\{r^2\mathrm{H}\left[(6+n)\frac{\mathrm{T}_2}{\mathrm{T}}+\frac{d\mathrm{T}_2}{dr}\Big/\frac{d\mathrm{T}}{dr}\right]\right\} \quad (15)$$

We notice that λ is a factor of the left-hand member of equation (14), but does not enter at all on the right-hand side. The equation may in fact be regarded as giving λ in terms of H, and on comparing coefficients of ρ_1/ρ we notice that the value of λ so given is of the order of

$$\frac{\frac{d}{dr}(r^2\mathrm{H})}{r^2\mathrm{P}} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Now $4\pi\frac{d}{dr}(r^2\mathrm{H})$ is of the order of magnitude of the rate at which energy is generated in a spherical shell of radius r and of unit thickness, while $4\pi r^2\mathrm{P}$ is of the order of the heat-energy in the same shell. Thus expression (16) is of the order of

$$\frac{\text{energy generated per unit time in star}}{\text{total heat energy of star}},$$

whence we see that $1/\lambda$ is comparable with the time in which the star would radiate away its store of heat at its present rate of radiation if no new heat were generated in its interior. For a Cepheid this is probably of the order of 100,000 years.

4. If we treated equation (12) in the same way, we should regard it as giving λ in terms of μ, and on comparing terms in r_1 (assuming the legitimacy of this for the moment) we should find that $1/\lambda$ was comparable with R^2/μ where R is the radius of the star. Now this is comparable with the time in which pulsations would be reduced to $1/e$ times their original amplitude under viscosity alone; I have estimated this to be of the order of something like 10^7 years (*M.N.*, 1925 June, p. 804), but the exact figure hardly matters. All that is essential for our present purpose is that the time is very much greater than the 100,000 years or so which we estimated as the value of $1/\lambda$ in the last paragraph, and this we may safely assume to be the case.

This makes it clear that our treatment of equation (12) has not been legitimate. The terms in μ are too small to give the correct value of λ by this method, the last term in equation (12) being negligible in comparison with the two remaining terms. It follows that in equation (11) also the last term is negligible in comparison with the remaining terms. Thus we may replace these two equations by

$$r_1(\lambda^2-q^2)=\left[\frac{4\pi\mathrm{M}_r}{r^3}+\left(\frac{2p}{\rho r}+\frac{1}{\rho}\frac{dp}{dr}\right)\frac{d}{dr}+\frac{p}{\rho}\frac{d^2}{dr^2}\right]r_1-\frac{1}{\rho}\frac{d}{dr}\left[(p+\tfrac{4}{3}a\mathrm{T}^4)\frac{\mathrm{T}_1}{\mathrm{T}}\right] \quad (17)$$

$$2\lambda q r_1=-\frac{\lambda}{\rho}\frac{d}{dr}\left[(p+\tfrac{4}{3}a\mathrm{T}^4)\frac{\mathrm{T}_2}{\mathrm{T}}\right] \quad . \quad . \quad . \quad (18)$$

5. The coefficient of viscosity μ has now disappeared entirely from our equations, so that the motion is the same as for a non-viscous star, and the rate of decay of the pulsations (measured by $1/\lambda$) arises solely from the flow of heat in the star. The time in which the pulsations decay to $1/e$ times their amplitude has been seen to be of the same order as the time in which the star would radiate away its present store of energy if this were not replenished, probably about 100,000 years for a Cepheid.

It is easy to find a physical reason for this. A consideration of physical dimensions shows that the time required for pulsations to diminish to $1/e$ times their value through flow of heat alone must be of the order of $\mathrm{R}^2\rho\mathrm{C}_v/\theta$, where θ denotes the coefficient of conduction of heat. The corresponding time of decay from viscosity is of the order of R^2/μ where μ is the kinetic coefficient of viscosity. By a well-known

formula of the Kinetic Theory of Gases, θ is approximately equal to $\mu\rho C_v$, so that the two times just mentioned would be about equal if the stars were spheres of gas in which heat was transferred by molecular collisions alone. In an actual star the transfer of heat by radiation far exceeds the transfer by molecular collisions; we may still speak of a "coefficient of conduction of heat," but its value is far higher than that for ordinary gaseous conduction. If θ is this coefficient, θ is now far greater than $\mu\rho C_v$, so that the time $R^2\rho C_v/\theta$ is much less than the time R^2/μ, which is the result previously obtained.

6. The period of pulsation $2\pi/q$, a few days at most, may be treated as insignificant in comparison with $1/\lambda$, which is of the order of 10^5 years. Regarding λ/q as a small quantity of the first order, all terms in H in our equation must also be regarded as being of the first order.

Neglecting small quantities of the second order, equations (15) and (17) reduce to

$$\rho C_v T_1 = \frac{P\rho_1}{\rho} \quad . \quad . \quad . \quad . \quad (19)$$

$$-q^2 r_1 = \left[\frac{4\pi M_r}{r^3} + \left(\frac{2p}{\rho r} + \frac{1}{\rho}\frac{dp}{dr}\right)\frac{d}{dr} + \frac{p}{\rho}\frac{d^2}{dr^2}\right] r_1 - \frac{1}{\rho}\frac{d}{dr}\left[\left(p + \tfrac{4}{3}aT^4\right)\frac{T_1}{T}\right] \quad (20)$$

If T_2 were zero, these would be the equations for a purely adiabatic oscillation. Using relation (19), equation (14) becomes

$$\rho C_v T_2 = -\frac{1}{r^2 q\lambda}\frac{d}{dr}\left\{r^2 H\left[(6+n)\frac{T_1}{T} + \frac{2r_1}{r} - \frac{2\rho_1}{\rho} - \frac{dr_1}{dr} + \frac{dT_1}{dr}\Big/\frac{dT}{dr}\right]\right\}$$

and the elimination of T_2 from this and equation (18) determines the complete oscillation, except for the boundary condition which must be satisfied at the surface of the star. For the discussion of the emitted radiation it is sufficient to neglect λT_2 entirely in comparison with T_1. The boundary condition may be taken to be

$$\sigma T_0^4 = \tfrac{1}{2}H,$$

where T_0 is the temperature at the star's surface, and this can be satisfied by a solution of the type under discussion.

Thus the solution which we have obtained is one in which, to a very close approximation, r, ρ, T, and H, and so also $4\pi r^2 H$, the total flow of energy, are all in the same phase. The phase difference between r (or ρ) and H is of the order of the stars period divided by 10^5 years, or, say 10^{-7} in radians.

7. The objections to the Shapley-Eddington pulsation theory which have been revealed by our analysis are :—

I. The luminosity of the star would be in phase with r, instead of with $\dot{r}$ as is observed.

II. The light-curve would show fore-and-aft symmetry, contrary to observation, and it is difficult to suppose that the inclusion of terms of second and higher orders in the displacements could modify this.

III. The pulsation would disappear altogether in a period of about 100,000 years, and this is too small to reconcile with the observed frequency of Cepheids in the sky.

To these may be added two other objections to which I have previously drawn attention :—

IV. We should expect to find oscillations of all possible amplitudes down to zero, whereas no Cepheids are known whose amplitude of oscillation is less than a certain limiting value.

V. We should expect to find oscillations of different and incommensurable frequencies in progress simultaneously in the same star, whereas the observed light-curves of Cepheids show a single definite period.

8. The next step in the problem is to examine to what extent the fission theory of Cepheid variables which I recently put forward * escapes these difficulties. According to this theory, a Cepheid variable is a star which is rotating in the state of secular instability which immediately precedes fission ; it is probably an approximately pear-shaped mass undergoing violent oscillations. Before secular instability sets in, any pulsations of the star must be damped in the way we have already considered, but as the star's rotation increases and the onset of instability is approached, the value of λ decreases and finally λ vanishes at the moment when instability sets in. At first, doubtless, λ will have a small positive value and the amplitude of the oscillation will increase exponentially. Shortly, however, a steady state will be reached in which dissipative forces prohibit any further increase of amplitude. From now on the oscillations are maintained at an approximately constant amplitude by energy drawn from the star's potential and rotational energy, but a secular change is in progress which ends in fission into two detailed masses.

I. The general equation for the internal oscillations of a rotating body is †

$$\frac{d}{dt}\left(\frac{\partial T_R}{\partial \dot{\theta}_s}\right)-\frac{\partial T_R}{\partial \theta_s}+\omega(\beta_{1s}\dot{\theta}_1+\beta_{2s}\dot{\theta}_2+\ \ldots)=-\frac{\partial}{\partial \theta_s}(W-\tfrac{1}{2}\omega^2 I)+F_s \qquad (21)$$

where θ_1, θ_2 . . . are the generalised co-ordinates of the motion relative to rotating axes, T_R its kinetic energy, W the potential energy, ω the angular velocity of rotation of the axes and of the body at the moment, I the moment of inertia, and F_1, F_2, . . . F_s . . . the externally applied forces.

On putting $\omega=0$ we pass to the equations for a pulsating non-rotating body. From these equations it is possible to deduce our equation (4), the viscosity term on the right representing the force F_s in equation (4). We have seen that equation (4) and its companion equations had a solution in which all the variables oscillated in approximately the same phase. Equation (21) shows that there can be no such solution when the gyroscopic terms in ω are taken into account, for these at once throw an equal-phase solution out of step. If ω is small, the solution will be of the form

$$r=r_0+r_1\cos qt,$$
$$\rho=\rho_0+\rho_1\cos qt+\rho_2\omega\sin qt,$$
$$T_1=T_0+T_1\cos qt+T_2\omega\sin qt.$$

* *M.N.*, 1925 June, p. 797.

† See, *e.g. Cosmogony and Stellar Dynamics*, p. 28.

This differs from our previous solution (i) by the inclusion of terms in sin qt of amplitude proportional to ω, and (ii) by the omission of the term $e^{-\lambda t}$ and terms in λ, the damping now being checked by the supply of energy from the star's rotation, etc. The emission of radiation, being proportional to the value of r^2T^4 at the star's surface, will no longer be in phase with r; it will contain terms proportional to cos qt, which are in phase with r, and also terms proportional to ω sin qt, which are in phase with $\dot{r}$. When ω is small, the latter terms disappear and the light-variation reduces to that already discussed. It is already known that the variations in r^2 and T^4 to a large extent neutralise one another, for an expansion and contraction which is homologous except close to the boundary of the star, the emission of radiation is proportional to r^{-n} where n is the small index in the opacity formula. Thus the total variation which arises from the terms in cos qt may well be slight, and when ω is large the terms in sin qt may well represent the greater part of the light-variation. In such a case we have the light-variation approximately in phase with $\dot{r}$, in agreement with observation.

II. Equation (21) does not retain its form when the sign of dt is changed, so that the light-curve from a single Cepheid ought not, in general, to show fore-and-aft symmetry. On the other hand, equation (21) retains its form when the signs of ω and dt are changed together. We should expect the signs of ω to occur at random, and so should expect statistical fore-and-aft symmetry in the light-curves, and this is not found. There may, I think, prove to be a real difficulty here; it is of course a new version of the old difficulty that the major axes always pointed in the same direction relative to the line of sight when Cepheids were interpreted as binary stars.

III. On the fission theory the oscillations are kept up to approximately constant amplitude by extraneous supplies of energy. The time for which the oscillations endure is determined simply by the length of time required for the completion of the process of fission. If we multiply equation (21) throughout by $\dot{\theta}_s$ and add corresponding sides of all the equations of the system, we obtain the well-known equation

$$\frac{d}{dt}(T_R+W-\tfrac{1}{2}\omega^2 I)=F_1\dot{\theta}_1+F_2\dot{\theta}_2+\ldots\ldots \quad (22)$$

Since F_s is the force on the co-ordinate θ_s arising from ordinary viscosity, the right-hand of this equation represents the rate at which viscous forces do work. The whole time of fission, then, is the time required for viscous forces to fritter away an amount of energy equal to the difference between the initial and final values of $T_R+W-\frac{1}{2}\omega^2 I$. The time required for viscous forces to fritter away the energy of a pulsation is, I have estimated, of the order of 10^7 years. Clearly the period of duration of fission must be several times longer, since the change in the energy $T_2+W-\frac{1}{2}\omega^2 I$ is several times the energy of a pulsation. At a guess we may perhaps suppose that a star remains a Cepheid for a period of the order of 10^8 or 10^9 years, which would probably be adequate to account for the observed frequency of Cepheids.

IV. On the fission theory we do not expect the oscillations to die

away; they remain of finite amplitude until they become transformed into the orbital motions of binary systems. Thus the circumstance that Cepheids of very small amplitude are unknown supports the fission theory.

V. On the fission theory, the oscillations are initially all of the same period, namely, that of the oscillation through which instability first sets in. At first both this period and that of the star's rotation ought to figure in superposition in the light-curve, but later on the two periods coincide and the light-curve ought to show a single distinct frequency.

9. It is of course not pretended that the analysis just given is in any way exhaustive or rigorous. It constitutes at best a rough preliminary survey which, so far as it goes, appears to favour the fission theory of Cepheid variation. The only formidable difficulty in the way of this theory appears to the writer to be that mentioned in II., namely, the statistical absence of fore-and-aft symmetry in the light-curve. But the feature is common to Cepheids and to most classes of long-period variables, and it seems permissible to hope that one of the many physical factors which have not been allowed for in our analysis may be found to explain it.

On the Kinetic Theory of a highly Ionised Gas. By E. Persico.

(*Communicated by Professor A. S. Eddington, F.R.S.*)

Although we have no experience of the properties of matter at as high a temperature as is supposed to exist in the interior of the stars, yet the study of its physical properties is made possible by the fortunate circumstance that all the difficulties arising for ordinary matter, from the complicated and imperfectly known law of action between molecules, at such temperatures disappear. It is well known that the Kinetic Theory of a gas has been completely worked out by the method of single collisions * on the hypothesis that molecules are centres of force attracting or repelling each other with a force depending on the distance, and, in particular, proportional to $\frac{1}{r^s}$. (By "the method of single collisions" we mean that it is assumed in these theories that the time during which a molecule is acted upon by another is short in comparison with its whole life, and the possibility of simultaneous action of more than one molecule is entirely disregarded.) Yet if we apply this theory, putting $s=2$, to the ionised gas, we find—as Gruner † pointed out—a paradoxical result, because some of the integrals coming into the formulæ for the phenomena depending on the mean free path—namely, viscosity, diffusion, heat and electricity conduction—become infinite when the exponent s is as small as 2. The physical reason is that when the

* See particularly Chapman, *Phil. Trans.*, 216 A (1915), p. 279, and 217 A (1916), p. 115.

† *Ann. d. Ph.*, **35**, 381, 1911.

intermolecular forces do not fall off rapidly with the distance, it happens *normally*, and not exceptionally, that a particle is simultaneously acted upon by several others, and therefore the single collisions hypothesis (in the meaning already explained) holds no longer. So the kinetic theory of a gas formed by positive nuclei and electrons, although simpler than that of an ordinary gas from the physical point of view (because the intermolecular forces are known), is much more difficult from the mathematical standpoint.

The rigorous treatment of the gas theory, without the single *collisions* assumption, is of course too complicated, but we shall try to take roughly into account the possibility of multiple collisions, in the following way :—

Around a given particle those of opposite sign will crowd, while those of the same sign will be repelled : this will give rise to an *induced* charge of opposite sign surrounding the particle, and diminishing its electrostatic action at a distance. Debye and Hückel,* dealing with the same problem in the case of a strong electrolyte, have assumed that this charge can be regarded as continuous, and, applying Boltzmann's theorem, have found the expression of the force—as a function of r—due to a particle and to its induced charge : this force of course falls off, for r tending to ∞, more rapidly than $\frac{1}{r^2}$. The same argument can be applied, with a modification suggested by Professor Eddington, to the case of the ionised gas. Now, if we put *this* law of force—instead of $\frac{1}{r^2}$—in the kinetic theory formulæ, we really take into account—though in an approximate way—the action of all the surrounding particles, besides that of the nearest one. In fact our integrals assume a finite value, and the above difficulty disappears.

It is not possible, however, to carry out the integration analytically, but, as an example, we shall assume some definite numerical conditions, and work out the solution by numerical integration.

The two fundamental integrals to be evaluated are (adopting Chapman's notations)

$$\Omega'_{12}(V)=4\pi V\int_0^\infty \sin^2\chi \,.\, pdp,$$

$$\Omega''_{12}(V)=\pi V\int_0^\infty \sin^2 2\chi \,.\, pdp,$$

where V is the relative initial velocity of the two colliding particles of masses m_1, m_2 ; 2χ is the angle through which this velocity is rotated after the collision ; and p is the perpendicular distance from a particle to the straight line which would be followed by the other, if there were no mutual action between them.

Four more integrals of this kind—namely, Ω'_{11}, Ω'_{22}, Ω''_{11}, Ω''_{22}—refer to collisions between particles of the same sign, and could be treated in the same way, but we do not need them. In fact it is easily understood, and a rough calculation verifies, that the phenomena we

* *Phys. Zeit.*, **24**, 185, 1923.

are dealing with are chiefly controlled by the motion of the electrons (the nuclei being very nearly stationary), and the free paths of the electrons are scarcely affected by collisions with other electrons, the collisions with the nuclei being much more frequent and more effective.

The deflection 2χ is related to the apsidal angle θ_0 by $2\chi=2\theta_0-\pi$, and θ_0 is given by

$$(1) \quad \theta_0(V, p)=\int_0^{\eta_0} \frac{d\eta}{\sqrt{1-\eta^2-\frac{2e_1}{V^2}\frac{m_1+m_2}{m_1m_2}\phi\left(\frac{p}{\eta}\right)}},$$

$\phi(r)$ being the potential at a distance r from a nucleus, and η_0 being the least positive root of the denominator. The suffix 1 refers to electrons, 2 to nuclei.

For the potential around a nucleus, Debye and Hückel give the expression

$$\phi=e_2\frac{e^{-\beta r}}{r}, \qquad \text{where} \qquad \beta^2=\frac{4\pi e_2{}^2 n_2}{RT}.$$

This corresponds to an induced negative charge, gradually decreasing outwards and tending to infinity at the centre. But Professor Eddington * pointed out that, in the astronomical problem, it is nearer to the truth to assume that the induced density is practically uniform (and equal to the average electron density) in the vicinity of the nucleus, and then decreases, according to Debye and Hückel's law, from a distance r_0 to infinity. Therefore we assume

$$(2) \quad \phi=-\frac{2\pi e_1 n_1}{3}r^2+\frac{e_2}{r}+\tilde{\phi}_2 \qquad \text{for } r<r_0,$$

$$(2') \quad \phi=C\frac{e^{-\beta r}}{r} \qquad \text{,, } r>r_0$$

(n_1 being the number of electrons per cubic centimetre, and e_1 the electron charge). The constants C and $\tilde{\phi}_2$ are to be determined so as to make ϕ and $\frac{d\phi}{dr}$ continuous for $r=r_0$; the value of r_0 is taken as the distance corresponding to $\phi=\frac{RT}{e_2}$. The condition of continuity of $\frac{d\phi}{dr}$ is

$$(3) \quad C\frac{e^{-\beta r_0}}{r_0}\left(\frac{1}{r_0}+\beta\right)=\frac{4\pi e_1 n_1}{3}r_0+\frac{e_2}{r_0{}^2}.$$

The condition of continuity of ϕ, and the condition imposed on r_0, give

$$(4) \quad -\frac{2\pi e_1 n_1}{3}r_0{}^2+\frac{e_2}{r_0}+\tilde{\phi}_2=\frac{RT}{e_2},$$

$$(5) \quad C\frac{e^{-\beta r_0}}{r_0}=\frac{RT}{e_2},$$

* *M.N.*, **86**, 1, 1925.

or, remembering the expression of β, and remarking that $e_1 n_1 = -e_2 n_2$ (because the gas is supposed to be electrically neutral),

$$(6) \quad . \quad . \quad . \quad 1+\beta r_0+\tfrac{1}{3}(\beta r_0)^2=\frac{e_2^2\beta}{RT}\frac{1}{\beta r_0}.$$

This is the equation determining βr_0, and therefore r_0. Then $\tilde{\phi}_2$ can be obtained from (4), and C from (5), and so ϕ is completely determined.

We must now assume some numerical values to carry out the calculation. We choose the following, corresponding roughly to a point of the Sun where the temperature is $\frac{2}{3}$ of that at the centre :—

Temperature, $T=22 \cdot 10^6$.
Density, $\rho=10$.
Number of free electrons per atom, 25.
Number per cubic centimetre, $n_1=3 \cdot 10^{24}$, $n_2=1{\cdot}2 \cdot 10^{23}$.
Mass, $m_1=8{\cdot}9 \cdot 10^{-28}$, $m_2=8{\cdot}5 \cdot 10^{-23}$.
Electric charge, $e_1=-4{\cdot}8 \cdot 10^{-10}$, $e_2=1{\cdot}2 \cdot 10^{-8}$.
Mean velocity of electrons (and, very nearly, mean relative velocity of collision),

$$\bar{V}=\sqrt{\frac{3RT}{m_1}}=3 \cdot 10^9.$$

With these numerical values we get $\beta=2{\cdot}7$, and the equation (6) becomes

$$1+x+\tfrac{1}{3}x^2=\frac{12}{x}$$

(where $x=\beta r_0$). Solving graphically we obtain $x=2{\cdot}3$, whence $r_0=0{\cdot}86 \cdot 10^{-8}$. From this we get $C=2{\cdot}1 \cdot 10^{-8}$ and $\tilde{\phi}_2=-1{\cdot}37$. So the function is completely determined. If we write u for $\frac{1}{r}$, and call y the expression appearing in (1), namely,

$$y(u)=\frac{2e_1}{V^2}\frac{m_1+m_2}{m_1 m_2}\phi(r),$$

we have

$$y(u)=\frac{33{\cdot}8 \cdot 10^{13}}{u^2}+{\cdot}135 \cdot 10^{-8}u-{\cdot}154 \qquad \text{for } u>1{\cdot}16 \cdot 10^{-8},$$

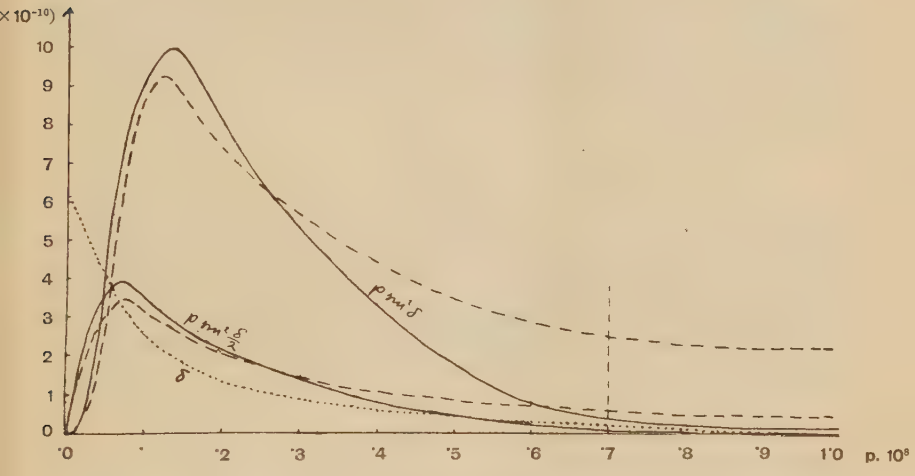
$$y(u)=2{\cdot}4 \cdot 10^{-9}ue^{-\frac{2{\cdot}7 \cdot 10^8}{u}} \qquad ,, \quad u<1{\cdot}16 \cdot 10^{-8}.$$

With these formulæ we have tabulated the values of $y(u)$ for $u=0$, ·1, ·2 . . . up to 1·3, and for 1·5, 1·7 . . . up to 7·1. For higher values of u the first term is negligible, and y becomes linear.

The next step is to calculate the integral (1), namely,

$$\theta_0=\int_0^{\eta_0}\frac{d\eta}{\sqrt{1-\eta^2+y\left(\frac{p}{\eta}\right)}},$$

whence the deflection 2χ is immediately obtained. The integration was performed numerically for the following values of p: 0, ·05, ·1, ·2, ·3, ·4, ·6, ·8, 1·0 ($\times 10^{-8}$). Simpson's method was employed from 0 up to a certain value of η: for the upper part of the integral, where the curve to be integrated becomes very steep, it was found better to approximate for the quantity $1-\eta^2+y$ with a linear or a quadratic function of $\eta-\eta_\theta$, and to integrate analytically. Other points were obtained by interpolation, and the results for δ were plotted as a curve in fig. 1. From this curve, values of the functions $p \sin^2 \chi$ and $p\pi \sin^2 2\chi$ were deduced. The diagram shows also the curves corresponding to the same functions, on the hypothesis of the ordinary inverse square law attraction (dotted lines).



Now, it is evident from the diagram that our curves tend to zero very quickly, and have a finite area, which, as above stated, is not the case for the dotted curves. Chapman * overcame the difficulty of the infinite integrals by neglecting the action of particles more distant than $d=\frac{1}{\sqrt[3]{n_1+n_2}}$ (average molecular distance): this corresponds to cutting off the dotted curves at the vertical dotted line: our result, as is seen, is of the same order of magnitude.

We can now apply our calculation to find the coefficient of diffusion of the electrons among the ions. This is given, to a first approximation, by †

$$D_{12}=\frac{3RT}{2\pi(n_1+n_2)m_1}\frac{1}{\frac{4}{\pi}\sqrt{\frac{\bar{h}m_1}{\pi}}\int_0^\infty e^{-hm_1V^2}\Omega'(V)dV}.$$

The integration with respect to V should be made numerically, but, for

* *M.N.*, **82**, 5, 1922.
† Chapman, *Phil. Trans.*, *loc. cit.*

our approximation, we can be content with the average value of V chosen above. Therefore we write

$$\int_0^\infty e^{-hm_1V^2}\Omega'(V)dV=\Omega'(\bar{V})\int_0^\infty e^{-hm_1V^2}dV$$

$$=\Omega'(\bar{V})\tfrac{1}{2}\sqrt{\frac{\pi}{hm_1}},$$

and

$$D_{12}=\frac{3RT}{4(n_1+n_2)m_1\Omega'(\bar{V})}.$$

The numerical integration of our curve $p\sin^2\chi$ gives for Ω'_{12} corresponding to $V=3\cdot10^9$ the value $\Omega'_{12}=4\cdot10^{-8}$. By substituting this value we find $D_{12}=21$.

The coefficient of viscosity is related to the coefficient of diffusion by the approximate formula *

$$\mu=\frac{\rho D_{12}}{2\left|\frac{e_1}{e_2}\right|},$$

which gives $\mu=4{\cdot}2$ as value of the viscosity in the interior of the Sun. For reference, it may be remarked that the viscosity of olive oil at ordinary temperature is about 1.

I am indebted to the International Education Board for a fellowship which has enabled me to pursue this investigation at Cambridge, and to Prof. A. S. Eddington for his interest in my work.

The Effect of Viscosity on the Oscillation of a Cepheid Variable.

By E. Persico.

(*Communicated by Professor A. S. Eddington, F.R.S.*)

If the pulsation theory of Cepheid variables is to be accepted, it has to be proved that any effect tending to damp the oscillation is quite negligible, so as to allow for the observed practically persistent oscillation. Professor Eddington † took into consideration the damping arising from radiation of heat from the hotter to the cooler parts of the star, and proved this to be sufficiently small. But, as it has been recently found that the viscosity in the interior of the stars is sometimes greater than that of oil, it remains to be examined whether the effect of such viscosity is also negligible. In fact it must be observed that, although the movement is radial, and therefore no friction of one layer on another (in the ordinary meaning) exists, yet the viscosity has some effect, because the particles of any layer, going by diffusion into the

* Chapman, *M.N.*, *loc. cit.*

† *M.N.*, 79, 2, 1918.

negihbouring one, which has a different radial velocity, and there assuming the average velocity, transform a part of the mechanical translational energy into energy of molecular agitation. This can be stated more rigorously and evaluated quantitatively in the following way.

We know from hydrodynamics that the energy transformed into heat in 1 c.c. per second is

$$\Phi=-\tfrac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)^2+$$
$$+2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2+\tfrac{1}{2}\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right)^2+\tfrac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+\tfrac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2\right]$$

where u, v, w are the components of the velocity. If the movement is radial, and $f(r)$ is the velocity, the latter expression becomes, as a simple calculation shows,

$$\Phi=\tfrac{4}{3}\mu\left(f'-\frac{f}{r}\right)^2.$$

We see that there *is* loss of energy, unless $f'=\frac{f}{r}$ or $f=r\times$ const.

Put $r=\frac{\mathrm{R}}{6{\cdot}9}\xi_0$ (where R is the radius in centimetres) and suppose * $r\xi_1\cos\frac{2\pi}{\tau}t$ be the radial displacement: then the velocity is

$$f=-\frac{2\pi}{\tau}r\xi_1\sin\frac{2\pi}{\tau}t,$$

whence

$$f'=-\frac{2\pi}{\tau}\left(r\frac{d\xi_1}{dr}+\xi_1\right)\sin\frac{2\pi}{\tau}t$$

and

$$\Phi=\tfrac{4}{3}\mu\left(\frac{2\pi}{\tau}r\frac{d\xi_1}{dr}\sin\frac{2\pi}{\tau}t\right)^2=\tfrac{4}{3}\mu\left(\frac{2\pi}{\tau}\xi_0\frac{d\xi_1}{d\xi_0}\sin\frac{2\pi}{\tau}t\right)^2.$$

Averaging over the time, and writing ξ_1' for $\frac{d\xi_1}{d\xi_0}$,

$$\overline{\Phi}=\frac{8\pi^2}{3\tau^2}\mu\xi_0{}^2\xi_1{}'^2.$$

The energy lost in the whole star in one period is

$$\mathrm{W}_l=\frac{8\pi^2}{3\tau}\int_0^{\mathrm{R}}\mu\xi_0{}^2\xi_1{}'^2 4\pi r^2dr=\frac{32\pi^3}{3\tau}\left(\frac{\mathrm{R}}{6{\cdot}9}\right)^3\int_0^{6{\cdot}9}\mu\xi_0{}^4\xi_1{}'^2d\xi_0.$$

Now assume μ to be proportional to $\mathrm{T}^{5/2}$ (T being the absolute temperature) and call μ_c, T_c the values of μ, T in the centre of the star, and $\tilde{\mathrm{T}}$ the temperature in a standard star: then

$$\mu=\mu_c\left(\frac{\mathrm{T}}{\mathrm{T}_c}\right)^{5/2}=\frac{\mu_c}{\tilde{\mathrm{T}}_c{}^{5/2}}\tilde{\mathrm{T}}^{5/2}$$

* These notations correspond to those employed in Eddington's paper.

and

$$W_l=\frac{32\pi^3}{3\tau}\left(\frac{R}{6\cdot9}\right)^3\frac{\mu_c}{\tilde{T}_c^{5/2}}\int_0^{6\cdot9}\tilde{T}^{5/2}\xi_0^4\xi_1'^2d\xi_0.$$

The energy of oscillation is $\frac{1}{2}\rho f^2_{\max.}$ per c.c., and in the whole star

$$W_0=\left(\frac{R}{6\cdot9}\right)^5\frac{8\pi^3}{\tau^2}\int_0^{6\cdot9}\rho\xi_0^4\xi_1^2d\xi_0,$$

or, introducing the central density ρ_c and the density in the standard star $\tilde{\rho}$,

$$W_0=\frac{8\pi^3}{\tau^2}\left(\frac{R}{6\cdot9}\right)^5\frac{\rho_c}{\tilde{\rho}_c}\int_0^{6\cdot9}\tilde{\rho}\xi_0^4\xi_1^2d\xi_0.$$

The ratio of these two energies is (taking $\tilde{T}_c=6\cdot59\,.\,10^6$, $\tilde{\rho}_c=\cdot108$)

$$\frac{W_l}{W_0}=\frac{4}{3}\frac{\tau\mu_c}{\left(\frac{R}{6\cdot9}\right)^2\frac{\rho_c}{\cdot108}(6\cdot59\,.\,10^6)^{5/2}}\frac{\int_0^{6\cdot9}\tilde{T}^{5/2}\xi_0^4\xi_1'^2d\xi_0}{\int_0^{6\cdot9}\tilde{\rho}\xi_0^4\xi_1^2d\xi_0}.$$

The two integrals cannot be evaluated up to $\xi_0=6\cdot9$ because the law of variation of ξ_2 in the surface layers is unknown, but as in these layers $\tilde{\rho}$ is very small, and $\tilde{T}$ drops very quickly to zero, we can safely stop our integration at $\xi=5\cdot5$ or so. In this way we get approximately $97\,.\,10^{15}$ for the top integral, and $1\cdot8$ for the lower.

Let us now apply this formula to the typical Cepheid δ Cephei, for which

$$\tau=4\cdot6\,.\,10^5 \qquad \rho_c=\cdot028 \qquad \frac{R}{6\cdot9}=2\cdot2\,.\,10^{11}.$$
$$T_c=5\cdot25\,.\,10^6$$

If we calculate the value of the viscosity μ corresponding to these conditions of temperature and pressure, by the method shown in a previous paper, we get $\mu_c=\cdot06$ approximately. By substituting in the last formula all these numerical values we obtain

$$\frac{W_l}{W_0}=1\cdot6\,.\,10^{-18},$$

so that the time of the decay of the pulsations is of the order 10^{18} secs. The rate of decay of the oscillations on account of viscosity is therefore small enough to escape observation.

The author is indebted to the International Education Board for a fellowship which enabled him to study these problems at Cambridge, and to Professor Eddington for his helpful advice.

Observations of (*Ch.* 1375) *X Persei in* 1907–25.

By A. N. Brown, M.A.

Some years ago X Persei, which is BD+30°591 (Harvard designation 034930), had acquired a remarkable reputation, the particulars of the variation being thus succinctly given by the late Miss Agnes Clerke in her *Problems in Astrophysics* (ed. 1903), p. 369: "At Potsdam, in 1898, M. M. Müller and Kempf noticed a star in Perseus as variable in an unprecedented fashion. After an indefinite term of constancy at 6·3 magnitude, it began in 1892 to decline at the very slow rate of one-eighth of a magnitude yearly, and continued to do so for six years. The counter-process was comparatively rapid. In twenty months the object had regained its former status, so that the complete oscillation occupied $7\frac{2}{3}$ years. This time, however, there was no long stationary maximum. Already by the end of 1899 fading had made some progress; but it remains to be seen whether any true periodicity can be established." And in the Harvard *Second Catalogue of Variable Stars* (1907), p. 35, a quotation from Müller and Kempf is given which states that X Persei is "a binary system, in which one star is at present transiting across the other. Period very long, perhaps 7·67 years."

Attracted by these statements, I began to observe the star towards the end of 1907, and it seems that the observations may be worth publishing, if only to show that, whatever the nature of the variation may have been before they began, during the years 1908–25 (1) the light did not remain constant for any considerable period, as is said to have been the case in 1888–92 (Müller and Hartwig's *Geschichte und Literatur des Lichtwechsels*, etc., i. 99), and (2) that no uninterrupted rise and fall extending over several years occurred.

After plotting the observations, it was found quite impossible to assign dates for maxima, but an attempt has been made to deal with the minima. It must be remarked, however, that from the beginning of May until early in July it is impracticable to observe the star owing to its position in the sky. Thus gaps occur in the observations at that period in every year. Before about the middle of 1917, it is difficult, if not impossible, to determine dates for minima, but from this time onwards the observations become more numerous, and seem to indicate the dates contained in the following table:—

Date.	Julian Date.	Mag.	Interval.
1918 Mar. 20	2421673	6·5	..
1919 Mar. 16	2034	6·4	361
1920 Feb. 15	2370	6·6	336
1921 Feb. 27	2748	6·5	378
1922 Apr. 16±	3161	6·6	413
1923 Mar. 11	3490	6·4	329
1924 Feb. 22	3838	6·6	363
1924 Mar. 23	3868	6·5	
1925 Feb. 13	4195	6·8	342

The fourth column above shows the interval in days from the last minimum. A mean has been taken between the two dates given for 1924. A minimum probably occurred in November 1925, but no date can be fixed for it until further observations have accumulated.

It appears that M. Luizet has hazarded the opinion that X Persei is a Long Period Variable, having a period of about 367 days (*Bulletin Astronomique*, xxxii. 239), an opinion which Herr Müller declares to be in direct contradiction to Luizet's own observations as well as those made at Potsdam. In this connection it is possibly worth noting that the mean period derived from the intervals in the above table is about 360 days.

The light at maximum is shown by my observations to vary from 6·0 to 6·3 m., the extreme range of the variation being 0·8 m. A star having such a small range of variability is clearly an unsuitable object for visual observation, and it is much to be hoped that a long and careful series of determinations with a photometer may be forthcoming in the future. It may be added that X Persei is contained in the first of the two valuable "Lists of Variable Stars needing Special Attention," compiled by Professor A. A. Nijland for the International Astronomical Union, in which it is described as being of the R Coronæ Borealis type.

The following table shows the comparison stars used :—

B.D.	Hagen's No.	H.C.O. Mag.	H.A.
+30° 582	2	6·22	50, 41
31 650	3	6·23	50, 41
31 649	4	6·51	54, 42
31 662	5	6·70	54, 43

The last column indicates the volume and page of the *Harvard Annals* from which the information given has been derived.

A few records giving identical results on consecutive nights having been omitted, there remain the 364 observations printed below. Two methods of comparison have been employed—the "fractional," the records of which are written in the form 2 (2) v (1) 5, in which the observer estimates by what fraction of the whole light interval between the comparison stars 2 and 5 the variable is fainter than 2 and brighter than 5. In employing the other method an attempt is made to estimate directly in tenths of a magnitude. Thus 2−3, 5+2 shows that the variable was considered ·3 m. fainter than star 2, and ·2 m. brighter than star 5. In all cases equal weight is given to each estimate. The observations were made with 5-inch, 3-inch, and 2-inch refractors, with powers varying from 16 to 50. A pair of binoculars was occasionally used in 1909.

The Observations.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1907			
Nov. 2	7882	2−0·5	6·3
4	7884	2−0·5, 3−0·5	6·3
15	7895	2+1, 3+1	6·1
29	7909	2−1·5, 3−1·5, 4+1	6·4
1908			
Jan. 4	7945	2+1, 3+1	6·1
12	7953	2−1, 3−1	6·3
Feb. 2	7974	2−1, 3−1	6·3
8	7980	2−1·5, 3−1	6·4
15	7987	2−2	6·4
22	7994	2−1, 3−0·5, 4+1	6·3
27	7999	2−1, 3−1	6·3
Mar. 9	8010	2−1, 3−0·5	6·3
28	8029	2−1, 3−0·5	6·3
Apr. 8	8040	2−1, 3+0·5	6·3
Oct. 3	8218	2+1	6·1
Nov. 7	8253	2+0·5, 3+1	6·2
17	8263	2+1, 3+1	6·1
19	8265	2−1, 3−1	6·3
23	8269	2−1, 3−1, 4+1·5	6·3
28	8274	2+0·5, 3+1	6·2
1909			
Jan. 8	8315	2−1, =3	6·3
19	8326	2−0·5, =3	6·3
26	8333	2−0·5, =3	6·3
Feb. 5	8343	2−1, 3−1	6·3
7	8345	2−1, 3−1	6·3
12	8350	2−1, 3−1	6·3
19	8357	2−2, 3(1)v(1)4	6·4
21	8359	2−2, 3(2)v(1)4	6·4
Apr. 1	8398	2−2, 3−2	6·4
8	8405	2−2, 3(2)v(1)4	6·4
10	8407	2−2, 2(1)v(2)5, 3(2)v(1)4	6·4
Aug. 23	8542	=2, =3	6·2
Sept. 8	8558	=2, =3	6·2
Sept. 18	8568	=2, =3	6·2
Oct. 6	8586	=2	6·2
8	8588	=2, =3	6·2
17	8597	=2, 3+1	6·2
25	8605	2+2, 3+3	6·0
29	8609	2+2, 3+2	6·0
Nov. 5	8616	2+1, 3+2	6·1
7	8618	2−0·5, 3+1	6·2
8	8619	2−1, =3	6·3
10	8621	2−0·5, =3	6·3
12	8623	2−0·5, 3+1	6·2
17	8628	2−1, 3+0·5	6·3
21	8632	=2	6·2
Dec. 8	8649	2−1, 3(1)v(2)4	6·3
29	8670	2−1, 3(1)v(3)4	6·3
31	8672	2−1·5, 3(1)v(3)4	6·3
1910			
Jan. 3	8675	2−1, 3−0·5	6·3
10	8682	2−0·5, =3	6·3
19	8691	2−0·5, =3	6·3
25	8697	2−0·5, =3	6·3
30	8702	2−1, 3−1	6·3
Feb. 3	8706	2−1, 3−1	6·3
9	8712	2−1·5	6·4
11	8714	2−2, 3−2	6·4
15	8718	2−1	6·3
22	8725	=2	6·2
Mar. 5	8736	2−2, 3−2	6·4
15	8746	=2, =3	6·2
22	8753	2+0·5	6·2
29	8760	2−1, 3−1	6·3
Apr. 1	8763	2−1	6·3
Sept. 12	8927	=2	6·2
Oct. 6	8951	=2	6·2
Nov. 1	8977	=2	6·2
11	8987	=2	6·2
20	8996	=2	6·2

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1910			
Dec. 19	9025	=2	6·2
22	9028	=2	6·2
29	9035	2+1	6·1
1911			
Jan. 2	9039	2+1	6·1
9	9046	2+1	6·1
30	9067	2(3)v(2)5	6·5
Feb. 19	9087	=2	6·2
20	9088	2−1·5	6·4
Mar. 7	9103	=2	6·2
Apr. 3	9130	2+1	6·1
Oct. 28	9338	2−2	6·4
Dec. 15	9386	2+1	6·1
20	9391	2+1	6·1
1912			
Jan. 27	9429	2+1, 3+1	6·1
Mar. 7	9469	2+1	6·1
10	9472	2+1·5	6·1
Oct. 7	9683	2+0·5	6·2
10	9686	2+2	6·0
14	9690	2+1	6·1
Dec. 29	9766	2−1	6·3
1913			
Jan. 5	9773	2−1	6·3
9	9777	2−1	6·3
12	9780	2−1	6·3
26	9794	2−1	6·3
Feb. 2	9801	2−1·5	6·4
22	9821	2−1	6·3
Mar. 11	9838	2−1	6·3
23	9850	2−2·5	6·5
Sept. 7	0018	2+1	6·1
Dec. 31	0133	=2	6·2
1914			
Feb. 15	0179	=2	6·2
22	0186	2−2	6·4
Mar. 21	0213	2−3	6·5
1915			
Jan. 19	0517	2−2, 3(1)v(1)4	6·4
Dec. 12	0844	=2	6·2
1916			
Jan. 22	0885	2−3, 5+2	6·5
Feb. 5	0899	2−3	6·5
Apr. 11	0965	2−2	6·4
Aug. 1	1077	2−2	6·4
Sept. 27	1134	2−1·5	6·4
Oct. 20	1157	=2	6·2
Nov. 26	1194	=2	6·2
1917			
Feb. 9	1269	2+1	6·1
14	1274	2−1	6·3
Mar. 14	1302	2−1	6·3
19	1307	2−2	6·4
22	1310	2−1·5	6·4
Apr. 23	1342	2+1	6·1
July 25	1435	=2	6·2
Aug. 18	1459	2+1	6·1
22	1463	2−1	6·3
Sept. 12	1484	=2	6·2
22	1494	2+1	6·1
Oct. 6	1508	2+1	6·1
Nov. 4	1537	=2	6·2
Dec. 6	1569	=2	6·2
1918			
Jan. 3	1597	=2	6·2
30	1624	=2	6·2
Feb. 24	1649	2−2	6·4
28	1653	2−1	6·3

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1918			
Mar. 8	1661	2−2, 5+3	6·4
12	1665	2−2, 5+2	6·5
16	1669	2(1)v(1)5	6·5
30	1683	2(1)v(1)5	6·5
Apr. 6	1690	2(2)v(3)5	6·4
14	1698	2−1, 5+3	6·4
July 16	1791	2+1	6·1
Aug. 10	1816	=2	6·2
14	1820	=2	6·2
Sept. 1	1838	=2	6·2
27	1864	2−1	6·3
Oct. 4	1871	2−1	6·3
7	1874	=2	6·2
Nov. 8	1906	2−1	6·3
13	1911	2−1	6·3
Dec. 12	1940	2−1·5	6·4
24	1952	=2	6·2
1919			
Jan. 7	1966	=2	6·2
17	1976	=2	6·2
Feb. 9	1999	=2	6·2
Mar. 2	2020	2−1	6·3
16	2034	2(2)v(3)5, 2−2	6·4
24	2042	2−2	6·4
27	2045	2(1)v(1)4	6·4
Apr. 2	2051	2−1, 3−1	6·3
20	2069	=2	6·2
Aug. 21	2192	=2	6·2
27	2198	=2	6·2
Sept. 6	2208	2+1	6·1
17	2219	=2	6·2
19	2221	=2	6·2
28	2230	=2	6·2
Oct. 3	2235	=2	6·2
16	2248	=2	6·2
26	2258	2−1	6·3
Nov. 11	2274	=2	6·2

Date.	Julian Date.	Comparisons	Ded. Mag.
1919			
Dec. 9	2302	2−1	6·3
31	2324	2−2	6·4
1920			
Jan. 5	2329	2−2, 3(1)v(3)5	6·4
16	2340	2(1)v(2)5	6·4
19	2343	2(3)v(2)5, 2−3	6·5
24	2348	2(3)v(1)5, 3−3·5	6·6
Feb. 6	2361	2(3)v(1)5	6·6
8	2363	2(3)v(1)5	6·6
11	2366	2(2)v(1)5	6·5
17	2372	2(3)v(1)5	6·6
Mar. 2	2386	2(3)v(1)5	6·6
7	2391	2(5)v(2)5	6·6
12	2396	2(1)v(2)5	6·4
18	2402	2(1)v(3)5	6·3
22	2406	2(1)v(3)5	6·3
31	2415	2(1)v(2)5	6·4
Apr. 12	2427	2(1)v(4)5, 2−1	6·3
16	2431	=2	6·2
21	2436	=2	6·2
28	2443	=2	6·2
May 3	2448	=2	6·2
Aug. 19	2556	=2	6·2
26	2563	=2	6·2
Sept. 5	2573	=2	6·2
10	2578	=2	6·2
12	2580	=2	6·2
18	2586	=2	6·2
19	2587	2−0·5	6·3
Oct. 11	2609	=2	6·2
13	2611	=2	6·2
20	2618	=2	6·2
23	2621	2−1	6·3
Nov. 1	2630	2−1	6·3
4	2633	2−2	6·4
11	2640	2−1	6·3
Dec. 1	2660	2+1	6·1

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1920			
Dec. 9	2668	=2	6·2
28	2687	=2	6·2
30	2689	=2	6·2
1921			
Jan. 12	2702	=2	6·2
31	2721	2−1	6·3
Feb. 12	2733	2−1	6·3
20	2741	2(1)v(2)5, 2−2	6·4
24	2745	2(1)v(2)5	6·4
26	2747	2(1)v(1)5	6·5
Mar. 1	2750	2(1)v(1)5	6·5
10	2759	2(1)v(3)5	6·3
18	2767	2(1)v(2)5	6·4
24	2773	2(1)v(3)5	6·3
Apr. 1	2781	2(1)v(2)5	6·4
4	2784	2(1)v(3)5	6·3
6	2786	2−2	6·4
Sept. 3	2936	2+1	6·1
7	2940	2+1	6·1
25	2958	=2	6·2
Oct. 1	2964	2+1	6·1
4	2967	2+1	6·1
8	2971	2+1	6·1
14	2977	2−1	6·3
16	2979	2−1	6·3
23	2986	2+1	6·1
Nov. 6	3000	2+1	6·1
8	3002	2+1	6·1
26	3020	2+0·5	6·2
Dec. 20	3044	2−1	6·3
25	3049	2−2	6·4
28	3052	2(1)v(3)5, 2−2	6·4
31	3055	2(1)v(3)5, 2−2	6·4
1922			
Jan. 4	3059	2(1)v(3)5, 2−2	6·4
10	3065	2(3)v(4)5	6·4
1922			
Jan. 16	3071	2(1)v(1)5	6·5
Feb. 9	3095	5+1	6·6
13	3099	2(4)v(1)5	6·6
17	3103	2(2)v(1)5	6·5
24	3110	2(3)v(2)5	6·5
26	3112	2(2)v(1)5	6·5
Mar. 2	3116	2(2)v(1)5	6·5
7	3121	2(2)v(1)5	6·5
9	3123	2(7)v(2)5	6·6
12	3126	2(3)v(1)5	6·6
20	3134	2(3)v(1)5	6·6
25	3139	2(3)v(1)5	6·6
27	3141	2(3)v(1)5	6·6
Apr. 4	3149	2(3)v(1)5	6·6
19	3164	2(5)v(2)5	6·6
Aug. 15	3282	2−2	6·4
Sept. 15	3313	2−1	6·3
29	3327	2(1)v(4)5, 2−1·5	6·3
Oct. 13	3341	2+1	6·1
15	3343	2+1	6·1
22	3350	2+1	6·1
28	3356	2+1	6·1
30	3358	2+1	6·1
Nov. 15	3374	2+1	6·1
23	3382	2+1	6·1
Dec. 18	3407	2+1	6·1
23	3412	2+1	6·1
27	3416	2+1	6·1
1923			
Jan. 3	3423	=2	6·2
9	3429	2+1	6·1
14	3434	2+1	6·1
22	3442	2+1·5	6·1
Feb. 4	3455	2−0·5	6·3
11	3462	2+1	6·1
Mar. 4	3483	2(2)v(3)5	6·4
17	3496	2(2)v(3)5	6·4

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1923			
Mar. 24	3503	2(1)v(3)5	6·3
Apr. 8	3518	2(1)v(3)5	6·3
13	3523	2−1	6·3
20	3530	2+1	6·1
26	3536	2+1	6·1
Sept. 5	3668	2−1	6·3
11	3674	2−1	6·3
15	3678	2−1	6·3
23	3686	2−1	6·3
Oct. 4	3697	2−1	6·3
14	3707	2+1	6·1
22	3715	2+1	6·1
Nov. 4	3728	2+1	6·1
10	3734	=2	6·2
14	3738	=2	6·2
30	3754	2+1	6·1
Dec. 3	3757	2+1	6·1
15	3769	2+1	6·1
19	3773	2+1	6·1
23	3777	2+1	6·1
1924			
Jan. 6	3791	2−1	6·3
25	3810	2+1	6·1
Feb. 1	3817	2−1	6·3
16	3832	2(1)v(1)5	6·5
20	3836	2(4)v(1)5, 3−4	6·6
26	3842	2(1)v(2)5	6·4
Mar. 2	3847	2(3)v(2)5	6·5
4	3849	2(1)v(2)5	6·4
8	3853	2(1)v(4)5	6·3
13	3858	2(1)v(2)5	6·4
19	3864	2(2)v(1)5	6·5
29	3874	2(2)v(1)5	6·5
Apr. 1	3877	2(3)v(2)5	6·5
4	3880	2(1)v(3)5	6·3
7	3883	2−1, 5+4	6·3
9	3885	2−1	6·3

Date.	Julian Date.	Comparisons.	Ded. Mag.
1924			
Apr. 14	3890	2(1)v(4)5	6·3
20	3896	2(1)v(5)5, 2−1	6·3
May 1	3907	2(1)v(4)5	6·3
July 11	3978	2−1	6·3
Aug. 7	4005	2−1	6·3
25	4023	=2	6·2
31	4029	=2	6·2
Sept. 9	4038	2−1	6·3
22	4051	2−2, 5+3·5	6·4
27	4056	2−1·5	6·4
Oct. 19	4078	2+1	6·1
23	4082	2−1	6·3
Nov. 3	4093	=2	6·2
17	4107	2−2	6·4
27	4117	2−1	6·3
Dec. 5	4125	2−1	6·3
14	4134	2+1	6·1
24	4144	2−1	6·3
1925			
Jan. 9	4160	=2	6·2
23	4174	2(1)v(2)5, 22−2	6·4
Feb. 1	4183	2(3)v(2)5	6·5
4	4186	2(1)v(1)5, 3−2, 4+1	6·4
15	4197	5−1	6·8
17	4199	2(5)v(1)5	6·6
22	4204	2(3)v(1)5	6·6
Mar. 2	4212	2(4)v(1)5	6·6
8	4218	2(4)v(1)5	6·6
12	4222	2(3)v(1)5	6·6
18	4228	2(3)v(2)5	6·5
21	4231	2(3)v(2)5	6·5
28	4238	2(3)v(2)5	6·5
Apr. 1	4242	2(3)v(2)5	6·5
3	4244	2(3)v(2)5	6·5
6	4247	2(3)v(2)5	6·5
11	4252	2(2)v(1)5	6·5

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.	Date.	Julian Date.	Comparisons.	Ded. Mag.
1925				1925			
Apr. 21	4262	2(3)v(1)5	6·6	Oct. 27	4451	2−1, 5+4	6·3
Aug. 11	4374	2(2)v(1)5	6·5	Nov 4	4459	2−1, 5+4	6·3
15	4378	2(2)v(1)5	6·5	8	4463	2(2)v(3)5	6·4
25	4388	2(1)v(2)5	6·4	15	4470	2−3, 5+2	6·5
Sept. 10	4404	2(1)v(3)5	6·3	25	4480	2−3, 5+2	6·5
14	4408	2(2)v(3)5	6·4	Dec. 2	4487	2−3, 5+2	6·5
21	4415	2(1)v(3)5	6·3	5	4490	2−3, 5+2	6·5
25	4419	2−1, 5+4	6·3	9	4494	2−3, 5+2	6·5
Oct. 8	4432	2−2, 5+3	6·4	11	4496	2−2, 5+3	6·4
14	4438	2−2, 5+3	6·4	17	4502	2−1, 5+4	6·3
23	4447	2−1, 5+4	6·3	30	4515	2−2, 5+3	6·4

Brackenhurst,
Bucklebury Common, Berks:
1926 *January* 1.

On a Period of approximately 9·2 years in the Greenwich Observations of Magnetic Declination and Horizontal Force. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. Analysis of earthquake records led to the recognition of a periodicity of 4 years in their occurrence, which was in the course of the work interpreted as arising from a bipolar effect circulating round the earth in 8 years. But an error in one set of computations was found which rendered this hypothesis no longer tenable. While, however, it was still on trial it was natural to inquire whether any 8-year effect was traceable in magnetic observations, and the answer was apparently in the affirmative. Thus the series of observations of range of Greenwich magnetic declination for the years 1841–77, collected and arranged by W. Ellis, F.R.S. (*Phil. Trans.*, 1880, **2**, 544), yielded the results shown in Table I. Four sets of 8 years exactly may be taken from them in two ways, either starting with 1841 or ending with 1877; and as a general check on the work both sets were reduced independently, keeping an equivalent initial year for both sets. Further, the results for each month were kept separate, but the change of initial date is in this case specially corrected below.

The unit for the coefficients of $\sin\theta$ and $\cos\theta$ is $0'{\cdot}025$, owing to the omission (for convenience) of a divisor 4. In the coefficient of the combined term (in the form $r\cos(\theta-\text{phase})$) it is $0'{\cdot}1$. Its mean value is about $0'{\cdot}5$.

Table I.

Analysis of Ranges of Greenwich Mag. Dec. for a Period of 8 Years.

	1841–72.					1846–77.				
	sin θ.	cos θ.	Coeff.	Phase.	Diff.	sin θ.	cos θ.	Coeff.	Phase.	Diff.
				°	°				°	°
Jan.	− 35	+16	9	294	+ 11	− 39	+27	12	325	+ 20
Feb.	− 13	+26	8	333	+ 54	− 24	+26	9	317	+ 16
Mar.	− 14	− 8	4	240	− 35	− 17	+12	5	305	+ 8
Apr.	− 37	−16	10	251	− 21	− 12	+15	4	321	+ 27
May	− 17	− 2	4	263	− 5	− 22	+26	9	321	+ 31
June	− 14	−12	5	230	− 34	− 20	−16	6	231	− 55
July	− 11	−26	7	203	− 57	− 15	−10	4	237	− 45
Aug.	− 8	−18	5	204	− 52	− 13	−21	6	211	− 67
Sept.	− 10	+11	3	316	+ 64	− 21	+ 7	5	288	+ 14
Oct.	− 6	+ 3	2	296	+ 47	− 18	+ 7	5	291	+ 20
Nov.	− 23	+ 4	6	260	+ 15	− 26	+ 3	6	277	+ 10
Dec.	− 19	− 8	5	247	+ 6	− 25	−16	7	237	− 26
Sum	−207	−30	..	262	±401	−252	+60	..	284	±337

2. The phases represent the position of maximum referred to the middle of the appropriate month as zero, and should thus decrease at the rate of $360°/8 \times 12 = 3°{\cdot}8$ per month. The differences from the mean progressively corrected in this way are shown in the columns headed Diff.

[For the actual corrections, see sixth column of Table II.]

The numerical sums of these residuals give as means of the errors $\pm 33°$ and $\pm 28°$; but they are clearly not altogether accidental—there is some tendency to be negative in the summer. None of them approaches 90°, however, and we may take the accordance as establishing a small periodicity of approximately 8 years.

3. The next step was to reduce a corresponding set of later date, taken from the Greenwich volumes. [Actually, before doing this, the horizontal force for the same years (1841–77) was examined for the 8-year term. Also a correction for the sunspot cycle was first applied, to see whether the difference between summer and winter would be reduced; but it was rather emphasised. Seeing, however, that the period was found to require a considerable correction, it seems useless to give the results of this work.] The correction for progressive change of initial date is explicitly shown in the sixth column of Table II.

4. It will be seen that we get harmonic terms with about the same magnitude of coefficient, but a quite different maximum phase—so different that without other evidence we should scarcely know whether there was an increase (from 262° in 1841–72 to 36° in 1873–1904) or

a decrease. But the two reductions in Table I. help us here. They show an increase of 22° in 5 years which in 32 years would be 141°; not far from 134°, which is the excess of 396° over 262°. The longer interval indicates an increase of 33·5° in 8 years or 4°·8 yearly travel, meaning that the true period is 8·82 years. But it is clear that 8 years is too far from the true period to give good results, and we, therefore, proceed to try 9 years.

TABLE II.

Analysis of Range of Gr. Dec. as in Table I. for the Years 1873–1904.

	sin θ.	cos θ.	Coeff.	Phase.	Corrn.	Corrd.	Difference from Mean.
				°	°		°
Jan.	+ 4	+ 18	5	13	−21	− 8	− 44
Feb.	+ 6	+ 18	5	18	−17	+ 1	− 35
Mar.	+13	+ 15	5	41	−13	+ 28	− 8
Apr.	+11	+ 10	4	48	−10	+ 38	+ 2
May	+ 9	+ 6	3	57	− 6	+ 51	+ 15
June	− 4	− 7	2	212	− 2	+210	+174
July	+ 6	− 5	2	130	+ 2	+132	+ 96
Aug.	+ 1	+ 3	1	18	+ 6	+ 24	− 12
Sept.	+15	+ 11	5	55	+10	+ 65	+ 29
Oct.	+16	+ 16	6	45	+13	+ 58	+ 22
Nov.	+ 7	+ 23	6	18	+17	+ 35	− 1
Dec.	+ 2	+ 10	2	11	+21	+ 32	− 4
Sum	+86	+118	..	36	..		±442

5. The ranges of Horizontal Force and of Declination for 1841–85, and again for 1868–1912 were accordingly arranged each in 5 sets of 9 years. They overlap, but it seemed better to take 45 years together so as to eliminate the sunspot cycle. The results are given in Tables III. and IV.

6. The unit of H.F. is that of the Greenwich Volumes, viz. 1γ. In taking the mean of the coefficients, they have first been multiplied by the cosine of the difference of phase from the mean, *i.e.* we form 11 cos 70°=+4, 12 cos 149°=−10, etc., so as to get the *effective* coefficients, which may occasionally be wrong in sign.

7. Here again the mean of the coefficients is not the actual mean (which has no physical significance), but the mean of the contributions to the mean phase, viz. 0′·57 cos 55°, 0′·27 cos 58°, etc., etc. Similarly, the mean phase is not found by taking the mean of the 12 phases, but that of the values of sin θ and cos θ, and forming tan θ_0 from them. It will be seen that the later observations in both cases show a smaller mean-of-the-errors; indeed the earlier observations of H.F. are very erratic—possibly due to faulty temperature corrections. We shall return to these results in §20.

TABLE III.

First Harmonic of Range of H.F. (Greenwich) in 9 Years.

	1841–85.			1868–1912.		
	Coeff.	Phase.	Diff.	Coeff.	Phase.	Diff.
		°	°		°	°
Jan.	11	78	− 70	14	197	+16
Feb.	12	359	−149	20	175	− 6
Mar.	9	132	− 16	9	200	+19
Apr.	12	149	+ 1	9	221	+40
May	31	175	+ 27	17	175	− 6
June	18	174	+ 26	6	168	−13
July	18	207	+ 59	17	199	+18
Aug.	29	179	+ 31	16	201	+20
Sept.	9	139	− 9	18	184	+ 3
Oct.	9	184	+ 36	20	240	+59
Nov.	4	281	+133	22	232	+51
Dec.	12	180	+ 32	16	221	+40
Mean	10	148	± 49	13	181	±24

TABLE IV.

First Harmonic of Range of Declination (Greenwich) in 9 Years.

	1841–85.			1868–1912.			
	Coeff.	Phase.	Diff.	Coeff.	Phase.	Diff.	Increase. 1841–68.
	′	°	°	′	°	°	°
Jan.	0·57	187	+55	0·37	169	+29	−18
Feb.	·27	190	+58	·37	136	− 4	−54
Mar.	·44	126	− 6	·61	167	+27	+41
Apr.	·46	123	− 9	·46	162	+22	+39
May	·31	134	+ 2	·42	145	+ 5	+11
June	·82	94	−38	·38	115	−25	+21
July	·44	93	−39	·28	105	−35	+12
Aug.	·58	108	+24	·51	153	+13	+45
Sept.	·35	140	+ 8	·64	180	+40	+40
Oct.	·41	154	+22	·67	191	+51	+37
Nov.	·47	212	+80	·53	168	+28	−44
Dec.	·64	180	+48	·53	168	+28	−12
Mean	·37	132	±32	·42	140	±26	+10

8. Since 1883 the Greenwich observations have been analysed in the form

$$V_t = M + a_1 \cos t + b_1 \sin t + a_2 \cos 2t + b_2 \sin 2t + \text{etc.},$$

where t represents the time from Greenwich mean midnight converted into arc at the rate of 15° to each hour, and V_t is the annual or monthly mean hourly value of the magnetic element at time t. From 1865 to 1882 the values of a_1 and b_1 were kindly calculated by Mr. D. J. R. Edney of the Royal Observatory. In transmitting these results to me, the Astronomer Royal wrote (under date 1923 May 8):—

"Before 1865 I am doubtful as to the reliability of the results, but am not certain."

The following analysis of the values of a_1 was carried out immediately on receipt of these valuable results; but attention was then called off to other matters and there has been regrettable delay in finishing the work till now.

9. The sunspot period is so near the suggested cycle of 9 years that it seemed desirable to remove it approximately before investigating the 9-year cycle.

For this purpose 11 years was assumed for the sunspot cycle. Perhaps 11½ years (as found in *M.N.*, **85**, 470–1) would have been better; but this work was done before the paper quoted had been written. From the standpoint of independent confirmation this is fortunate, as the double period of 23 years was suggested independently in the magnetic work (though under the disguise of a 24-year period). There are some points of interest about the presentation of this double period, so that, although in the light of the later work the magnetic reductions would be conducted differently, the historical account is worth giving briefly.

10. The values of a_1 for the month of January for the years 1865–1919 were first arranged in 5 sets of 11 years as follows:—

TABLE V.

$-a_1$: *Eleven-year Cycle for January: Unit* 0·01.

1865	288	217	157	170	217	265	255	272	349	244	143
1876	135	131	118	105	118	166	194	177	180	176	234
1887	175	162	107	151	122	198	224	170	204	274	173
1898	152	155	150	76	90	112	153	182	113	177	165
1909	200	172	160	106	110	94	140	184	238	218	232
Mean	190	167	138	122	131	167	193	197	217	218	189

The mean values were then subtracted from each member of the column above them, rearranged in 9 years, and each set of 9 years analysed harmonically. The same was done for February, March, and April. At this point it was thought well to review the results before proceeding farther. Table VI. gives the maximum phase of an angle θ which runs through its cycle in 9 years, as deduced from the successive groups.

11. The adopted zero points for the months are the middles of the first month, and hence not quite the same. To reconcile them we should add successive multiples of 3°·3; but it will be seen that this theoretical

TABLE VI.

Maxima of 9-year Cycles in a_1.

	1865.	1874.	1883.	1892.	1901.	1910.	Mean.
	°	°	°	°	°	°	°
Jan.	283	292	208	404	335	347	316
Feb.	217	290	252	429	260	360	311
Mar.	223	316	264	390	294	305	294
Apr.	210	302	196	458	254	329	271
Mean	233	300	230	420	286	335	
Corrected	233	187	230	307	286	222	
Period	233	178	212	280	250	177	
Resid.	+11	−44	−10	+58	+28	−45	

value does not suffice. Leaving this point for the moment, the striking feature is the alternation of large and small values. The mean for the 1874 cycle exceeds that of the 1865 and 1883 cycles by 69° : and the 1892 cycle exceeds its neighbours by 162°. The odd cycles 1883 and 1901, which have two neighbours, are in defect of them by 130° and 92°. The mean of these four values is 113°.

12. The application of this difference of 113° to the 1874, 1892, and 1910 columns does not, however, satisfactorily reconcile the means, which are shown in the line "Corrected." Discussing them for linear increase we find a value +9° for the increase per cycle, indicating a period of about 9·22 years. The line "Period" shows the effect of applying this correction : the clustering round the mean value 222° is now fairly good.

13. But clearly before proceeding farther it is desirable to identify the cause of the alternation in the columns if possible. It suggests a periodicity of about double the length of the 9 year. As a first approximation a period of 19 years was assumed. This gave successive maxima for January at 81°, 159°, 234°, which increase by 76°. The period thus indicated is

$$19\times 360°/284°=24\cdot1 \text{ years.}$$

But this value cannot be accepted as final, since the trial period of 19 years is so much in error. Let us now try 24 years.

14. The available material extended from 1865–1920 : two cycles of 24 years and 8 years over. We may thus get two cycles in either of two ways :—

A : 1865–1888 and 1889–1912,
or B : 1873–1896 and 1897–1920.

Both ways were used and the mean taken. The maxima shown by these 24 means (2 for each month) are all in the first quadrant (*i.e.* $\sin\theta$ and $\cos\theta$ both positive). [When the separate values A and B are regarded, among the 96 values 5 are small negatives and 1 is zero : these exceptional values being for June, July, and August, but it seems needless to give the details.] They are shown in Table VII.

TABLE VII.

Maxima for Two Successive Cycles of 24 Years.

Month.	(1) sin θ.	cos θ.	(2) sin θ.	cos θ.	(1) Max.	(2) Max.	(2)–(1).
					°	°	°
Jan.	+30	+20	+16	+17	57	43	−14
Feb.	+39	+36	+20	+16	47	50	+ 3
Mar.	+47	+26	+16	+36	61	24	−37
Apr.	+29	+31	+25	+22	43	49	+ 6
May	+14	+21	+20	+17	35	49	+14
June	+ 6	+12	+23	+ 7	27	73	+46
July	+12	+ 2	+24	+12	81	64	−17
Aug.	+15	+12	+21	+19	51	47	− 4
Sept.	+30	+32	+29	+28	44	47	+ 3
Oct.	+24	+27	+25	+32	42	38	− 4
Nov.	+26	+22	+10	+24	49	22	−27
Dec.	+13	+18	+14	+18	36	38	+ 2

15. It will be seen that 6 of the months show a decrease and 6 an increase, the mean being a decrease of 3°. If the period is 23 years there ought to be a decrease of 15°. It seems, however, highly improbable that the period differs from the double of the sunspot cycle: for casual errors may easily affect the maxima of two successive cycles. As an experiment the N. and S. latitudes and N. and S. log areas as quoted in *M.N.*, **85**, 467, were subjected to analysis for 2 sets of 24 years. The results were:—

	(1) Max.	(2) Max.	(2)–(1).
	°	°	°
N lats.	167	193	+ 26
S lats.	169	223	+ 54
N log areas	225	331	+106
S log areas	169	188	+ 19

The consistent testimony of these four elements from the Sun himself is in favour of a period longer than 24 years: and yet we need not suppose any real departure from 23 years; for the testimony of Wolf's numbers is in the other direction. Three sets of 23 years of Wolf's numbers, beginning with 1842, 1865, and 1888, give maxima at 81°, 77°, 51° respectively; showing a slight *decrease*, or a period shorter than 23 years. We may safely regard all these departures as accidental: they suggest, indeed, that in this matter of periodicity our magnetic observations, from their greater accuracy and their greater number, are able to give us better information than the Sun himself.

16. Before leaving the 24 or 23 year period it is well to remark that besides the half-period of 12 or $11\frac{1}{2}$ years (the sunspot period) the third harmonic of about 8 years is sensible, and unless care be taken will

affect any determination of a period near 9 years. We can see this for the Range of Declination by adding together the results of Table I. (1841–72) and Table II. (1873–1904), making a total of 64 years (1841–1904)=7×9 years+1 year: in which periods near 7 and near 9 years should both be eliminated. The sums for $\sin\theta$ and $\cos\theta$ are given in Table VIII.

TABLE VIII.

Eight-Year Period in Magnetic Dec. 1841–1904.

	$\sin\theta$.	$\cos\theta$.	Coeff.	Max.	C_1.	$O-C_1$.	C_2.	$O-C_1-C_2$.
				°	°	°	°	°
Jan.	−31	+34	5	318	317	+ 1	− 4	+ 5
Feb.	− 7	+44	6	351	313	+ 38	+ 13	+25
Mar.	− 1	+ 7	1	352	309	+ 43	+ 34	+ 9
Apr.	−26	− 6	3	257	305	− 48	+ 17	−65
May	− 8	+ 4	1	296	302	− 6	− 42	+36
June	−18	−19	3	224	298	− 74	−101	+27
July	− 5	−31	4	189	294	−105	−100	− 5
Aug.	− 7	−15	2	205	290	− 85	− 37	−48
Sept.	+ 5	+22	3	373	287	+ 86	+ 46	+40
Oct.	+10	+19	3	388	283	+105	+ 87	+18
Nov.	−16	+27	4	329	279	+ 50	+ 66	−16
Dec.	−17	+ 2	2	277	275	+ 2	+ 21	−19
Mean	−10	+ 7						

17. The unit for the coefficients is 0′·1. The column C_1 shows a correction for reducing consecutive months to the same zero: but the column $O-C_1$ clearly has an annual swing. The column C_2 therefore represents the expression

$$C_2 = -35^\circ \sin\theta + 48^\circ \cos\theta + 16^\circ \sin 2\theta - 52^\circ \cos 2\theta$$
$$= 59^\circ \cos(\theta - 324^\circ) + 54^\circ \cos 2(\theta - 82^\circ).$$

The residuals are fairly good, but the hypothesis of a term like C_2 is in need of other evidence before it can be accepted finally. Moreover, the coefficients shown in the fourth column are small.

18. To test this 8-year term further the coefficient a_1 was examined. The available years are now only 56 (1865 to 1920), but this should remove almost all the 9-year term. The results are shown in Table IX.

19. The column C_1 as before corrects the successive months to zero: and

$$C_2 = -23^\circ \sin\theta + 3^\circ \cos\theta + 12^\circ \sin\theta + 12^\circ \cos\theta$$
$$= 23^\circ \cos(\theta - 277^\circ) + 17^\circ \cos 2(\theta - 22^\circ).$$

We can obtain this from the C_2 of §17 by multiplying the coefficients by about 0·4, and adding 45° to θ. This fact lends some support to the otherwise rather dubious feature in the declinations. But there is a serious objection to regarding it as a character of this 8-year term,

TABLE IX.

Eight-Year Period in a_1 : 1865–1920 : *Unit* 0·01.

	sin θ.	cos θ.	Coeff.	Max.	C_1.	O–C_1.	C_2.	O–C_1–C_2.
				°	°	°	°	°
Jan.	−13	+ 2	13	279	248	+ 31	+15	+16
Feb.	− 7	+ 1	7	278	244	+ 34	+ 7	+27
Mar.	− 2	−13	13	189	240	− 51	−15	−36
Apr.	−14	+ 3	14	283	236	+ 47	−35	+82
May	+ 7	− 4	8	119	233	−114	−37	−77
June	− 4	− 7	8	209	229	− 20	−19	− 1
July	− 2	− 3	4	214	225	− 11	+ 9	−20
Aug.	−10	− 6	12	239	221	+ 18	+25	− 7
Sept.	− 7	− 9	11	218	218	0	+23	−23
Oct.	− 4	− 5	6	219	214	+ 5	+11	− 6
Nov.	− 4	− 3	5	233	210	+ 23	+ 5	+18
Dec.	−10	− 6	12	239	206	+ 33	+11	+22

viz. that though the results of Table I. show features very like those of Table VIII., those of Table II. do not.

20. Hence in trying the effect of removing the 8-year term it seems better to ignore any possible annual swing. Briefly we may compare the mean results of Tables I. and II. thus :—

	Table I. (1841–72).			Table II. (1873–1904).			II.–I.
	sin θ.	cos θ.	Max.	sin θ.	cos θ.	Max.	
			°			°	°
Means	−17	− 3	262	+ 7	+10	36	134
Subtract 8-year term.	−10	+ 7		−10	+ 7		
	− 7	−10	215	+17	+ 3	80	225

The advance of maximum is, by the subtraction of the 8-year term, increased from 134° to 225° in 32 years; which indicates a period of 9·48 years. We may now return to Table IV. where an advance of 8° in 27 years is found, indicating 9·06 years. In Table III., for H.F. an advance of 33° is found in the same interval: period 9·28 years. In §12 a period 9·22 years was deduced from the values of a_1 for January–April. These four values are in fair accord and the mean of all four is 9·26 years.

21. But we must now complete Table VI. by reducing the other months as in Table X., the figures of Table VI. being repeated for convenience.

22. The means in the last column but one are formed by adding the sines and cosines, not by taking the mean of the phases, which are sometimes ambiguous. It will be seen that the rapid drop in the first four months, noticed in §11, is not maintained. The mean for January to April is 298°, and for September to December is 265°; a drop of 33° in 8 months or 4°·1 per month, according fairly well with

TABLE X.

Maxima of 9-Year cycles in $-a_1$ (see Table VI.).

	1865.	1874.	1883.	1892.	1901.	1910.	Mean.	Corrected.
	°	°	°	°	°	°	°	°
Jan.	283	292	208	404	335	347	316	316
Feb.	217	290	252	429	260	360	311	314
Mar.	223	316	264	390	294	305	294	301
Apr.	210	302	196	458	254	329	271	281
May	108	312	309	408	243	303	310	323
June	150	225	99	373	55	279	301	318
July	132	304	73	425	279	290	359	379
Aug.	178	219	116	476	209	296	236	259
Sept.	191	225	283	407	268	295	285	312
Oct.	225	280	272	438	299	308	272	302
Nov.	183	312	257	489	273	293	249	282
Dec.	187	248	218	430	158	319	254	291
Mean	191	277	209	428	244	310		
Corrected	191	140	209	291	244	173		
Period	191	131	191	264	208	128		
Resid.	+5	−55	+5	+78	+22	−58		

the theoretical value 3°·3, due to the shift of adopted zero by 360°/9×12 per month. The values corrected for this theoretical drop of 3°·3 per month are shown in the last column. The individual phases in the table were assigned, where there is ambiguity, by reference to the last column, remembering the alternation of big and small noticed in Table VI. and fully apparent in Table X. also.

23. By the crude method adopted for Table VI. the excesses of 1874 and 1892 over their neighbours are 77° and 201°: and the defects of 1883 and 1901 are 144° and 125°: mean of the four 137°. The line "Corrected" shows an increase of 9° per cycle, determined from the observations. This is just about what we got from the first four months alone in §12, and indicates a period of 9·22 years. When allowance is made for this 9° advance, as in the line marked "Period," the mean value is 186°: and the residuals in the last line are very similar to those of Table VI.

24. The reductions for $-b_1$, were carried out precisely as those for $-a_1$, and Table XI. corresponds to Table X.

25. In Table XI. there are several cases of ambiguity which make it undesirable to take means of the phases, as was possible in Table X. The means at the foot of the Table as well as those in the last column were obtained by summing the sines and cosines separately. The well-marked alternation of Table X. has disappeared, or would have to be rather artificially supplied.

26. An attempt was made to deal with the cause of the alternation in Table X. by subtracting the 9-year term (approximately) and considering the residuals for some other periodicity. This seemed to be

TABLE XI.

Maxima of 9-Year Cycles in $-b_1$.

	1865.	1874.	1883.	1892.	1901.	1910.	Mean.
	°	°	°	°	°	°	°
Jan.	246	114	398	396	283	387	339
Feb.	237	315	416	365	283	355	303
Mar.	237	315	449	303	289	450	283
Apr.	222	103	374	360	288	335	290
May	222	196	196	437	291	345	279
June	192	151	411	423	295	314	253
July	193	123	487	424	253	333	187
Aug.	215	196	360	401	299	334	260
Sept.	240	184	332	354	290	401	267
Oct.	208	129	421	324	252	369	226
Nov.	198	92	343	313	240	385	249
Dec.	205	77	302	413	261	119	215
Mean	216	125	377	371	278	270	

possibly a small third harmonic of the sunspot period (about 7·7 years); and it was hoped that on now subtracting this from the original numbers the 9-year term might emerge more clearly. But nothing of great value was found.

27. The fact is that the exact period is made difficult of determination, because the available series of observations for a_1 (1865–1920) is scarcely long enough to separate it clearly from the 7·7-year term (and perhaps from other harmonics of the sunspot period). The coefficient for a_1 is about 10 of our units or 0′·10, and is of course small; but as the series of observations extends, the term should emerge more clearly. It does not seem as yet worth while to scrutinise more closely the other coefficients, even b_1, which shows more vagaries than a_1.

28. An attempt to find this 9·2-year period in sunspots, described in another paper, failed.

Summary.

Magnetic observations of ranges of declination and of horizontal force at Greenwich, for the years 1841 to 1904, show periodic variations in 9·26 years (see § 20). The period cannot be assigned with very great accuracy owing to the interference of the sunspot period of 23 years (approximately) with its second and third harmonics, the former being usually recognised as the chief cycle. The coefficient in range of declination is about 0′·4; and in range of horizontal force about 12γ.

The coefficients a_1 and b_1 in the expressions calculated for harmonic variation in the Greenwich volumes also show this 9·2-year period; and are similarly perturbed by the 23-year period and its harmonics. Apparently this 9·2-year period is not represented in sunspots (see a separate paper) and should have some other origin. Can it be connected with the revolution of the Moon's nodes?

On an Unsuccessful Search for the 9·2-Year Magnetic Period in Sunspot Records, with a New Analysis of those Records back to 1610. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In the previous paper evidence is adduced for the existence of a periodicity of 9·2 years in magnetic elements (declination, horizontal force, the coefficients a_1 and b_1, of the Greenwich reductions). The origin of such a periodicity might be sought in various directions: the movement of the Moon's perigee, for instance, has a period not far away. But magnetic elements vary so sympathetically with solar activity, and the double sunspot period of 23 years (to which attention has recently been called) presented itself so directly as a perturbation of the 9·2-year period, that it was natural to inquire whether the origin might not be there.

2. Accordingly Wolf's Sunspot numbers were arranged in sets of 9 years. In the first instance the discussion was confined to the years 1841–1920, covered by the magnetic observations; but the results obtained prompted the continuation of the series backwards to 1750. The use of 1840 as initial year instead of 1749 in the following table is merely a consequence of starting from the modern end of the series.

3. In Table I. are given the coefficients a and b of a term $a \sin\theta + b\cos\theta$, representing the first harmonic for each consecutive 9 years beginning with the initial date of the first column. To eliminate the effects of the 23 and $11\frac{1}{2}$-year terms, five consecutive sets are then taken together. The unit is that of Wolf's Sunspot numbers.

4. The mean values of a and b for five consecutive terms (45 years in all) are then converted into

$$a\sin\theta + b\cos\theta = r\cos(\theta - \theta_0),$$

so that θ_0 is the maximum value of θ. If the true period is 9·2 years, instead of 9·0 as assumed, the maximum will advance by $0{\cdot}2\times 360^\circ/9 = 8^\circ$ every cycle. The corresponding subtraction is made in the last two columns, and it will be seen that the maximum remains fairly steady except that

(*a*) For the four terms bracketed, it has been necessary to subtract 180°. The coefficient is reversed in sign;

(*b*) One term adjacent to the brackets, and one at the end, are anomalous.

5. If we accept the reversal (*a*) as a real phenomenon (*e.g.* due to the interference of two adjacent periodicities), anomalies such as (*b*) may arise from the process of combining together (in the mean of five) cycles from positive and negative groups. The reversal seems to take place in one direction or the other, about every five or six terms or 45 to 50 years, the double change taking 90 or 100 years. Taking the latter value, the interfering periodicities would be of 9·62 and 8·78 years, since, approximately,

$$100 = 10{\cdot}4\times 9{\cdot}62 = 11{\cdot}4\times 8{\cdot}78.$$

TABLE I.

First Harmonic in 9 Years of Wolf's Sunspot Numbers.

Initial Year.	a.	b.	Fives. a.	Fives. b.	r.	θ_0.	Corrn.	θ_1.
1750	+ 7·8	+28·9						
1759	+28·4	+ 7·1				°	°	°
1768	+45·7	+ 7·8	+37·8	+13·5	40·0	70	− 47	+23
1777	+58·2	+21·8	+32·5	+ 9·4	33·7	74	− 55	+19
1786	+48·8	+ 2·2	+29·6	+11·7	31·7	68	− 63	+ 5
1795	−18·5	+ 8·0	+22·6	+ 7·3	23·7	73	− 71	+ 2
1804	+14·0	+18·9	+ 4·5	+ 0·6	4·6	83	− 79	+ 4
1813	+10·4	−14·4	−17·0	− 3·8	17·4	257	− 87	(−10)
1822	−32·0	−11·8	−21·3	− 0·1	21·3	270	− 95	(− 5)
1831	−58·2	−19·8	−18·1	0·0	18·1	270	−103	(−13)
1840	−40·4	+26·4	−13·7	+ 3·0	14·0	283	−111	(− 8)
1849	+30·0	+19·8	− 2·5	− 5·0	5·2	207	−119	+88
1858	+32·4	+ 0·5	+ 2·5	− 2·5	3·6	135	−127	+ 8
1867	+24·0	−51·9	+ 5·2	− 2·6	5·7	116	−135	−19
1876	−32·9	− 7·3	+ 3·4	− 2·7	4·3	129	−143	−14
1885	−27·3	+26·2	+ 0·4	− 6·2	6·3	176	−151	+25
1894	+21·0	+19·1	− 9·7	− 3·3	10·2	251	−159	+92
1903	+17·8	−17·1						
1912	−27·1	−37·3						

6. But there is also a striking decay in the coefficient r, which has the value 30 or 40 in 1768, about 20 in 1820, and from 1850 onwards is a few units only; so that its modern coefficient scarcely seems large enough to be responsible for the term found in the magnetic records.

7. After trying several other expedients for testing the 9-year cycle directly, it was determined to attempt the removal of the 11-year cycle as completely as possible, so that the residuals might perhaps give clearer information. The best value of the main cycle was believed to be 11·5 years, with a sensible difference between alternate cycles which is physically important, since it may be connected with the curious reversal of magnetic polarity at minimum (see *M.N.*, **85**, 467). Hence the first and second harmonics for every 23 years were formed as in Table II., and then combined in pairs of 46 years to eliminate any periodicity near 9 years.

8. Looking first at the values of r_1, θ_1 (deduced from consecutive pairs of a_1, b_1), it is clear that the present hypothesis does not give us any consistent 23-year term. The phase from 1850 is the reverse of that before. The 11·5-year term shows a decreasing phase and the coefficient r_2 an increase. As a crude expedient the 92 years 1764–1855 were taken together, and again the 92 years 1833–1924. Subtracting them from the numbers in each set the mean values of the 23-year and 11·5-year periodicities for that set, the results were analysed in sets

TABLE II.

First and Second Harmonics for every 23 Years.

Initial Year.	a_1.	b_1.	r_1.	θ_1.	a_2.	b_2.	r_2.	θ_2.
				°				°
1749	−13	+13	..	..	−11	0		
1772	− 8	− 9	11	281	− 5	+33	18	334
1795	+ 8	0	5	180	−20	+ 5	22	327
1818	−31	+ 1	12	273	−34	−17	28	258
1841	+10	− 2	11	266	−37	+ 3	37	259
1864	+19	+ 2	15	90	−31	+17	35	287
1887	+ 8	+ 1	14	83	−22	+ 5	28	293
Means	− 1	+ 1	..	..	−23	+ 7	24	287

of 9 years as in Table III. The years 1833–1855 are common to both sets, and the means of the two results for these years have been used in Table III.

9. In column c_1 a theoretical advance of 4° per cycle (corresponding to a period of 9·1 years) is shown: and the quantities $\theta_0 - c_1$ would be reasonably small, if we are allowed two sets of reversal by 180°. But a more satisfactory solution is shown by column c_2, viz. an advance

TABLE III.

Wolf's Number in 9-Year Cycles after correcting for 23-Year and 11·5-Year Periodicities in two Parts.

Initial Year.	a.	b.	θ_0.	c_1.	$\theta_0 - c_1$.	c_2.	$\theta_0 - c_2$.	r.
			°	°	°	°	°	
1770	−10·2	+30·0	341	340	+ 1	289	+52	32
79	−21·3	+42·0	333	344	− 11	332	+ 1	47
88	+23·1	+41·1	29	348	+ 41	15	+ 6	47
97	− 1·1	+15·5	356	352	+ 4	58	−62	16
1806	+15·8	+ 6·2	68	356	+ 72	101	−33	17
15	+ 0·2	−20·0	180	0	+180	144	+36	20
24	−30·0	−36·2	220	4	+216	187	+33	47
33	+ 2·9	−31·1	175	8	+167	230	−55	31
42	−30·2	−29·5	226	12	+214	273	−47	42
51	+ 7·7	+10·7	36	16	+ 20	316	+80	13
60	− 1·8	+ 4·4	338	20	− 42	359	−21	5
69	+15·3	+ 8·0	62	24	+ 38	42	+20	17
78	+ 9·5	+ 7·1	53	28	+ 25	85	−32	12
87	+ 8·4	−11·8	144	32	+112	128	+16	14
96	− 3·1	− 6·4	205	36	+169	171	+34	7
1905	− 5·6	− 7·5	216	40	+176	214	+ 2	9
14	− 8·9	− 9·1	224	44	+180	257	−33	13

of 43° per cycle, corresponding to a period greater than 10 years. It is clear that this is the dominating cycle, and though there was still open the possibility that removing it might disclose a period near 9 years, previously submerged, the remainders gave no such indication when trial was made.

10. The search for a 9·2-year period in Sunspots might therefore have been abandoned at this point; but it seemed desirable to obtain as decisive evidence as possible, and therefore the investigation was pursued farther.

A list of maxima and minima back to 1611 was given by Wolfer in *V.J.S. der Naturf. Ges. in Zurich*, **47**, 240, 1902, and reproduced by Stratton in his recent *Astronomical Physics*, Appendix VII., but without the weights. It is so short that it may be reproduced here.

TABLE IV.

Wolfer's Sunspot Maxima and Minima.

Minima.		Maxima.		Minima.		Maxima.		Minima.		Maxima.	
Date.	Wt.	Date.	Wt.	Date.	Wt.	Date.	Wt.	Date.	Wt.	Date.	Wt.
1610·8	5	1615·5	2	1723·5	2	1727·5	4	1833·9	10	1837·2	10
1619·0	1	1625·0	5	1734·0	2	1738·7	2	1843·5	10	1848·1	10
1634·0	2	1639·5	2	1745·0	2	1750·3	7	1856·0	10	1860·1	10
1645·0	5	1649·0	1	1755·2	9	1761·5	7	1867·2	10	1870·6	10
1655·0	1	1660·0	1	1766·5	5	1769·7	8	1878·9	10	1883·9	10
1666·0	2	1675·0	2	1775·5	7	1778·4	5	1889·6	10	1894·1	10
1679·5	2	1685·0	2	1784·7	4	1788·1	4	1901·7	10	1906·4	10
1689·5	2	1693·0	1	1798·3	9	1805·2	5	1913·4	10	1917·6	10
1698·0	1	1705·5	4	1810·6	8	1816·4	8				
1712·0	3	1718·2	6	1823·3	10	1829·9	10				

To utilise this table the value 100 was assigned to each maximum, zero to each minimum, and straight lines were drawn between them. The values for the middle of the year were read off from these lines. The results were then analysed in sets of 9 years. The individual results are clearly affected by the 11-year period and need not be given: but sums of 5 cycles (45 years) ought to eliminate that periodicity and are given in Table V.

11. Now the first feature of the results to strike the attention is the steady character of the periodicity from 1610–1734—more than a century—as shown in the columns c_1 and $\theta_0 - c_1$. The residuals $\theta_0 - c_1$ are remarkably small until we reach mean date 1734. The previous mean date 1725 comprises the 45 years 1707–51: so that this steady periodicity lasts approximately up to the time when the sunspot numbers commence. Of course it thereby comes under suspicion of representing merely a change in recording, but the suspicion would have been better founded if it had referred to an irregularity instead of a remarkable regularity. The period is 8·75 years; and the mean maximum may be put at 1675·9.

TABLE V.

Nine-year Cycle in Wolfer's Maxima and Minima.

Mean Date.	Mean Coeffs. *a.*	Mean Coeffs. *b.*	R.	θ_0.	c_1.	θ_0-c_1.	c_2.	θ_0-c_2.
				°	°	°		
1626	−10·9	−10·9	15·4	225	224	+ 1		
1635	−10·6	−14·2	17·7	217	214	+ 3		
1644	− 4·3	− 9·9	10·8	203	204	− 1		
1653	− 3·1	−16·2	16·5	188	194	− 6		
1662	− 4·3	−17·3	17·6	194	184	+ 10		
1671	+ 1·5	−17·5	17·5	175	174	+ 1		
1680	+ 3·1	−18·0	18·2	170	164	+ 6		
1689	+ 4·7	−16·6	16·9	164	154	+ 10		
1698	+ 6·2	−11·2	13·2	151	144	+ 7		
1707	+12·4	− 5·6	13·5	114	134	− 20		
1716	+ 9·0	− 5·0	10·3	119	124	− 5	250	−131
1725	+ 7·1	− 4·0	8·1	120	114	+ 6	264	−144
1734	+ 1·4	− 6·3	6·4	168	104	+ 64	278	−110
1743	− 6·9	+ 0·8	7·0	277	94	+183	292	− 15
1752	−15·5	+10·9	18·9	305	84	−139	306	− 1
1761	−15·3	+26·2	30·3	330	74	−104	320	+ 10
1770	− 1·6	+35·1	35·1	357	64	− 67	334	+ 23
1779	− 3·4	+29·2	29·4	353			348	+ 5
1788	− 5·1	+ 2·7	5·8	297	..	..	2	− 65
1797	+ 4·1	+20·4	20·8	12	..	..	16	− 4
1806	+ 2·4	+ 5·7	6·1	23	183	−160	30	− 7
1815	− 2·6	− 8·4	8·8	197	190	+ 7	44	+153
1824	− 1·9	−11·5	11·6	189	197	− 8	58	+131
1833	− 3·7	−12·2	12·8	197	204	− 7	72	+125
1842	− 6·0	− 9·3	11·1	212	211	+ 1		
1851	+ 2·1	− 2·6	3·3	130	218	− 88	139	− 9
1860	− 2·9	− 1·6	3·3	241	..	..	219	+ 12
1869	− 3·9	+ 2·9	4·9	307	..	..	299	+ 8
1878	+ 0·2	+ 3·0	3·0	4	..	..	19	− 15
1887	+ 2·5	− 0·2	2·5	94	..	..	99	− 5

12. If we may trust these records, something quite new comes in about 1750. Both the coefficient R (which suddenly increases, after falling off) and the epoch of maximum θ_0 show this clearly. The maximum, instead of decreasing 10° per cycle, increases by 14° as shown

in the columns c_2 and $\theta_0 - c_2$. We can assign the group with mean date 1725 to c_1, and the group with mean date 1743 to c_2; so that a fair date for the change would be 1734. The period indicated by an increase of 14° per cycle is 9·35 years. It lasts till the cycle with mean date 1806, when there is an obvious discontinuity, the subsequent march being an increase of 7° per cycle or a period of 9·18 years, as shown by the figures below the heavy line in column c_1. About 1846 there is a new discontinuity, the increase being now 80° per cycle as shown below the heavy lines in column c_2. Such an increase would accumulate to $5 \times 80° = 400°$ in five cycles, *i.e.* more than a complete period of 9 years. It is therefore not surprising that the coefficient R should be small; and since $360° + 80°$ correspond to 11 years, we interpret these figures to mean that from about 1845 the 9-year period is so small that it is overwhelmed by the 11-year period even under the disadvantageous conditions (to the latter) of breaking up 45 years into 5 parts.

13. Such is the kind of interpretation which might be put upon these figures: but for the latter part of the table we have already obtained a more probable result in Table II. from the sunspot numbers (which are clearly preferable to the crude representation by lines joining maxima and minima), viz., that there is an advance of 42° per cycle. This would amount to 210° in 5 cycles, so that the periodicity would be much diluted and liable to interference from others. Hence the former analysis is to be preferred. It seems desirable, however, to examine afresh the figures derived from maxima and minima only for this 10-year period, if only to test their general trustworthiness during the seventeenth century, when we have nothing else. But first we may try to remove the 11-year period as best we can. If it is of constant length, the long series 1610–1920 should give us the length with considerable accuracy.

14. The process adopted was as follows: Removing the 8·75-year period from the years 1611 to 1743 the remainders were analysed in 12 years, and a drop of 23° per cycle found, indicating a period of about 11·2 years. The mean maximum, which should be determined with fair accuracy from all this material, in spite of its accidental errors, may be put at 1671·84. Now at the end of Table III. are a few maxima not much disturbed by other periodicities. If we simply take the mean of the last 5, viz. 1894·52, the difference $1894{\cdot}52 - 1671{\cdot}84 = 222{\cdot}68$ represents 20 cycles of 11·134 years. But the early maximum refers to the first harmonic, the later to the whole curve which is known to have a sharper rise than fall. If we find the maximum of the first harmonic for the later observations, we get the date 1895·2, increasing the difference for 20 periods to 223·4 years and the single period therefore to 11·17 years. We may take 67/6 years as being pretty near the mean value, to a first approximation.

15. The whole series from 1608 to 1920 was therefore arranged in sets of 11, with an extra year in 1641, 1708, 1775, 1842, 1909 (once every 67 years), and means were formed of the whole 28 sets as follows:—

37	27	20	25	41	59	72	79	74	65	52	(38)
35	24	21	27	41	57	71	79	77	67	51	(*c*),

the last term in brackets being the mean of the 5 excluded years. Harmonic analysis of the first 11 terms gives the second line (*c*) representing the expression

$$50-25\sin\theta-15\cos\theta=50+29\cos(\theta-239^\circ).$$

The representation is clearly very close.

16. It is scarcely to be expected that the coefficient should have an appearance of constancy in the middle of the series, which is known to be disturbed. But it is important to know whether it has a secular decrease. The means of the first 67 years are

32	20	14	23	39	58	74	82	75	67	54

and of the last 67,

32	18	9	18	41	64	79	87	81	67	52

These numbers are closely similar. Harmonic analysis gives

$$\text{First } 67 \quad 49-28\sin\theta-17\cos\theta=49+33\cos(\theta-239^\circ)$$
$$\text{Last } 67 \quad 50-31\sin\theta-20\cos\theta=50+37\cos(\theta-237^\circ).$$

17. Hence it was determined to try the general effect of assuming this periodicity constant throughout. The mean value of each column was subtracted from the numbers above it as a sufficiently accurate process for exploration. The residuals were then arranged in 9 years, and showed the 8·75-year period already detected quite clearly up to about 1750; but after that they were quite inconsistent with it. In the course of some attempts to suit them, it was noticed that from 1767 they clearly indicated a period close to 13 years. The mean values of twelve sets of 13 years from 1767 to 1922 are as in the top line :—

28	23	19	10	2	3	10	16	19	27	33	33	34
30	23	16	9	5	5	7	13	21	27	33	36	35
−2	0	+3	+1	−3	−2	+3	+3	−2	0	0	−3	−1

The second line is calculated from the formula

$$20-12\sin\theta+10\cos\theta=20+16\cos(\theta-310^\circ).$$

18. The separate periods give accordant results, with one or two exceptions, as follows :—

Table VI.

Initial Year.	θ_0.	θ_0-310°.	R.	R_0.	Initial Year.	θ_0.	θ_0-310°.	R.	R_0.
	°	°				°	°		
1767	345	+ 35	42	34	1845	44	(+94)	34	− 2
1780	262	− 48	59	40	1858	0	+50	21	3
1793	258	− 52	38	24	1871	345	+35	12	10
1806	260	− 50	17	11	1884	307	− 3	11	11
1819	321	+ 11	26	25	1897	309	− 1	20	20
1832	105	(+155)	20	−18	1910	270	−40	15	12

The column R shows the coefficient deduced direct : but not all of this is effective ; we must form $R_0=R\cos(\theta_0-310^\circ)$ to get the effective component. There is an appearance of decay in the column R_0, but this may not be real.

19. But it was clearly desirable to examine the earlier years for traces of this periodicity; and on doing so it was found to exist from the first, though its value became much enhanced about 1750. Each cycle of 13 years was analysed separately, but to eliminate other periodicities (especially the 8·75 one already detected) five cycles were taken together. The phases θ_0 in Table VII. show both a periodic swing (in ten cycles) and a steady change. To detect the latter, advances for 10 cycles were formed giving a mean of $-35°$ or a drop of $3°{\cdot}5$ per cycle. This makes the mean period less than 13 years by $3{\cdot}5\times 13/360 = {\cdot}12$ years, *i.e.* 12·88 years for the period. The representation of the drop is given in column c_1; and for the fluctuation in column c_2, where ϕ runs through its period in 10 terms and

$$c_2 = 29° - 60° \sin\phi - 44° \cos\phi.$$

The final columns R_1 and R_2 represent the effective parts of R_0 on the two suppositions, *i.e.* $R_1 = R_0 \cos(\theta_0 - c_1)$ and $R_2 = R_0 \cos(\theta_0 - c_1 - c_2)$.

TABLE VII.

Thirteen-year Term. Means of Five Cycles.

Initial Year.	θ_0.	c_1.	$\theta_0 - c_1$.	c_2.	$\theta_0 - c_1 - c_2$.	R_0.	R_1.	R_2.
	°	°	°	°	°			
1611	351	345	+ 6	− 15	+21	7	+ 7	+ 7
24	318	341	− 23	− 42	+19	9	+ 8	+ 9
37	313	338	− 25	− 42	+17	10	+ 9	+10
50	306	334	− 28	− 14	−14	9	+ 8	+ 9
63	347	331	+ 16	+ 30	−14	6	+ 6	+ 6
76	370	327	+ 43	+ 73	−30	4	+ 3	+ 4
89	421	324	+ 97	+100	− 3	6	− 1	+ 6
1702	479	320	+159	+100	+59	2	− 2	+ 1
15	336	317	+ 19	+ 72	−53	6	+ 6	+ 4
28	340	313	+ 27	+ 28	− 1	20	+18	+20
41	277	310	− 33	− 15	−18	24	+20	+23
54	274	306	− 32	− 42	+10	26	+22	+25
67	286	303	− 17	− 42	+25	27	+26	+24
80	267	299	− 32	− 14	−18	21	+27	+20
93	315	296	+ 19	+ 30	−11	7	+ 7	+ 7
1806	369	292	+ 77	+ 73	+ 4	12	+ 3	+12
19	378	289	+ 89	+100	−11	15	+ 1	+15
32	388	285	+103	+100	+ 3	13	− 3	+13
45	357	282	+ 75	+ 72	+ 3	15	+ 4	+15
58	321	278	+ 43	+ 28	+15	13	+10	+13

20. It becomes important to examine whether the fluctuation of maximum denoted by c_2 is real. Since 5 cycles have been taken together, periods differing sensibly from 13 years must have been eliminated. But we may express the fluctuation by the interference of two terms both near 13 years, of which one has a coefficient a consider-

ably larger than that of the other b. The period 12·88 years is that of the a term; and denoting the combination by

$$a\cos\theta+b\cos(\theta-\phi)=(a+b\cos\phi)\cos\theta+b\sin\phi\sin\theta$$
$$=R_2\cos(\theta-\chi)$$

then the coefficient R_2 varies between limits $(a+b)$ and $(a-b)$, while the values of χ, given by $\tan\chi=b\sin\phi/(a+b\cos\phi)$, vary between limits given by $\cos\phi=-b/a$, $\sin\chi=\pm b/a$. The column c_2 gives extreme values of χ as $+100°$ and $-42°$, or $+29°\pm71°$; so that $b/a=0·95$. The limits $(a+b)$ and $(a-b)$ for R should thus be in the ratio 1·95 to 0·05 or 39 to 1. This is much greater than any ratio suggested by the column R_2 in Table VI., especially in the later years, where R_2 ranges from 7 to 25, so that the ratio could not exceed 4 to 1, corresponding to $\sin\chi=0·6$ or $\chi=37°$. It must be remembered, however, that we have made a very artificial assumption about the ranges (making them uniformly 0 to 100) which may easily affect the coefficients of all the periodicities. We can only treat the results obtained as hints for trial, and we now return to Wolf's Sunspot numbers to try the effect of this fluctuating 13-year term.

21. Wolf's numbers were accordingly arranged in sets of 13 years; and then the initial year was shifted along the cycle as suggested by c_1+c_2 of Table VI. The means of the columns were as follows:—

25 35 40 54 59 60 65 58 51 45 38 30 27;

analysis of which gives

$$45+7·5\sin\theta-16·8\cos\theta=45+18·4\cos(\theta-159°).$$

The harmonic term was then subtracted from the columns, and the remainders restored to their proper sequence. They made 15 sets of $11\frac{1}{6}$ years: and the means of the 3 sets of 5 cycles are:—

68	62	49	42	42	43	48	45	47	57	56	(91)
60	54	43	37	23	13	11	19	42	72	78	(119)
57	47	36	29	26	23	21	24	41	67	63	(51)

Harmonic analysis of the three sets, treated as 11 years simply (without taking account of the three omitted terms in brackets), gives

1749–1803; $52-2·1\sin\theta+8·1\cos\theta=52+8·4\cos(\theta-346°)$
1804–1858; $41-6·3\sin\theta+25·6\cos\theta=41+26·4\cos(\theta-346°)$
1859–1913; $39-7·4\sin\theta+16·7\cos\theta=39+18·3\cos(\theta-336°)$.

22. Let us compare with these figures the corresponding deductions from Wolf's numbers uncorrected for this fluctuating 13-year term:—

55	47	36	38	45	53	61	60	60	65	58
58	52	40	33	20	11	11	21	46	74	80
75	63	46	32	20	11	6	11	31	67	71

with the corresponding analysis—

1749–1803; $53-10·4\sin\theta-1·0\cos\theta=53+10·5\cos(\theta-264°)$
1804–1858; $41-9·3\sin\theta+24·0\cos\theta=41+25·7\cos(\theta-339°)$
1859–1913; $39-2·3\sin\theta+29·8\cos\theta=39+29·9\cos(\theta-356°)$

23. It will be seen that the phases (346°, 346°, 336°), of the former set, corrected for the 13-year periodicity, are much more accordant than those of the second set (264°, 339°, 356°), unless we allow for an advance in the latter of some 45° per 5 cycles, or 9° per cycle; or a correction to period of 0·28 years; making it nearly 11·5 years as has been assumed in §§ 7-9. But we have seen that this value of the period does not agree with that assigned (§ 14) by the long stretch of maxima and minima 1610–1920. A further objection, and possibly a much more serious one from the physical standpoint, is that the double period of 23 years, which represents the difference between alternate cycles, is not consistently maintained in Wolf's uncorrected numbers. Now let us examine this feature with some care. In each set of 5 cycles we can form the mean of Nos. 1, 3, 5 for comparison with the mean of Nos. 2 and 4. The differences for the three sets give the following harmonic terms :—

TABLE VIII.

Differences between Alternate Cycles of 11·17 *Years for Corrected Numbers.*

1749–1803	$-14{\cdot}7 \sin\theta + 12{\cdot}0 \cos\theta = 19{\cdot}0 \cos(\theta - 309^\circ)$	$+24^\circ$
1804–1858	$-15{\cdot}5 \sin\theta - 3{\cdot}1 \cos\theta = 15{\cdot}8 \cos(\theta - 259)$	-26
1859–1913	$-10{\cdot}7 \sin\theta + 2{\cdot}2 \cos\theta = 11{\cdot}0 \cos(\theta - 282)$	-3
Mean	$-13{\cdot}6 \sin\theta + 3{\cdot}7 \cos\theta = 14{\cdot}1 \cos(\theta - 285)$	± 53

24. These figures show a difference between alternate cycles consistently maintained back to 1749—a much more reassuring result than that of Table II. We may compare the corresponding figures for the uncorrected Wolf numbers, viz. :—

TABLE IX.

Differences between Alternate Cycles for Uncorrected Numbers.

1749–1803	$-9{\cdot}8 \sin\theta + 14{\cdot}0 \cos\theta = 17{\cdot}1 \cos(\theta - 325^\circ)$	$+39^\circ$
1804–1858	$-7{\cdot}6 \sin\theta - 6{\cdot}7 \cos\theta = 10{\cdot}1 \cos(\theta - 228)$	-58
1859–1913	$-13{\cdot}1 \sin\theta + 1{\cdot}6 \cos\theta = 13{\cdot}2 \cos(\theta - 277)$	-9
Mean	$-10{\cdot}2 \sin\theta + 3{\cdot}0 \cos\theta = 10{\cdot}6 \cos(\theta - 286)$	± 106

The coefficients are smaller and the phases less consistent. The differences of the phases from those deduced from the mean values are shown in the last column, with the numerical sums; and the sum for the uncorrected numbers is just double of that for the corrected.

25. Hence recognition of the variable 13-year periodicity seems to bring the following definite advantages :—

(*a*) The period of 11·17 years, indicated by the long series of maxima and minima (1610–1920), is disclosed, whereas Wolf's numbers, uncorrected for the 13-year periodicity, suggest the longer period of 11·5 years.

(*b*) The shorter period carries with it a consistent difference between alternate cycles right back to 1749, which the longer period fails to do (as shown in Table II.).

(*c*) This difference (which may be of physical importance as connected with the change of magnetic polarity at minimum found at Mount Wilson) is shown more clearly and consistently by the numbers after correction for the 13-year periodicity.

26. It may be well to give the actual figures for the alternate cycles. Those given above were derived in sets of 3 and 2, from the 3 sets of 5 cycles. We shall now take the 15 cycles together forming 8 "odd" cycles beginning with 1749, and 7 "even" cycles; and we shall prefer the "corrected" numbers.

The 8 "odd" sets gave

63 53 39 28 18 21 31 35 48 67 63 —

and the 7 "even" sets—

60 56 47 45 44 33 23 23 38 63 70 (87),

the (87) being the mean of the 3 extra years which all come in "even" sets. The differences odd—even are

+3 −3 −8 −17 −26 −12 +8 +12 +10 +4 −7.

27. We will now look for the 10-year period suggested by Table III. Subtracting from the "odd" sets the first set of means, and from the "even" sets the second set, and arranging the residuals in 10 years, analysis gives the results of Table X.

TABLE X.

Ten-Year Period, after removing 13-*Year and* 11-*Year.*

Initial Date.	a.	b.	θ_0.	θ_c.	$\theta_0-\theta_c$.	r_0.	r_1.	r_c.	r_1-r_c.
			°	°	°				
1749	+29·0	+25·4	48	20	+28	38·6	34·1	40·5	− 6·4
1759	+23·8	+15·4	57	18	+39	28·4	22·1	37·3	−15·2
1769	− 6·0	+55·6	344	17	−33	55·9	46·9	34·1	+14·8
1779	−38·4	+48·0	322	15	−53	61·7	37·1	31·1	+ 6·0
1789	+ 5·2	+39·8	7	13	− 6	40·0	39·8	28·3	+11·5
1799	+10·2	+12·0	40	11	+29	15·7	13·7	25·8	−12·1
1809	+12·6	+ 7·2	60	10	+50	14·4	9·3	23·5	−14·2
1819	− 5·4	+11·6	335	8	−33	12·7	10·6	21·5	−10·9
1829	−40·6	+26·4	303	7	−64	48·4	21·2	19·6	+ 1·6
1839	− 5·2	+25·4	348	5	−17	25·9	24·8	17·9	+ 6·9
1849	+15·2	+ 8·8	60	3	+57	17·2	9·4	16·3	− 6·9
1859	− 2·6	+15·4	351	1	−10	15·6	15·4	14·9	+ 0·5
1869	+25·4	+22·2	49	0	+49	33·8	22·2	13·5	+ 8·7
1879	− 6·0	− 1·2	259	358	−99	6·1	−0·9	12·3	−13·2
1889	+14·4	+ 2·4	81	356	+85	14·6	12·7	11·3	+ 1·4
1899	− 6·6	+14·8	336	355	−19	16·2	15·3	10·5	+ 4·8

The notation is much as before: θ_0 is the maximum with an exactly 10-year period: θ_c represents a calculated maximum for a period of 9·96 years. It will be seen that only one of the differences $\theta_0-\theta_c$ exceeds

90°, though the next is very near 90°. The coefficient r_0 is found from $r_0^2=a^2+b^2$: but only part of it is effective, viz. $r_1=r_0\cos(\theta_0-\theta_c)$. The values of r_1 show a decay of some kind, which is compared with a systematic decay represented in the column r_c, each term being the same fraction of the one preceding.

28. Finally we may look for the periodicity near 8·75 years, which is strongly represented in the early years 1610–1730 as we saw in Table V. The result was of no great interest. If the term exists it is small and variable: and its maxima do not accord with those in the seventeenth century. We get, for instance, maxima at 1750·9 and 1785·4, which when combined with 1675·9 (see § 11) give $9\times8{\cdot}33$ years and $13\times8{\cdot}42$ years. Nothing suggests a 9·2-year period; and we must conclude that the magnetic period of this length does not originate in solar influence.

Summary.

1. Search for the 9·2-year magnetic period in the Sunspot variation has resulted in failure to detect it.

2. To leave no stone unturned, Wolfer's early records of maxima and minima (from 1610 onwards) were examined. These show clearly two periodicities, one of 11·2 years and one of 8·75 years (Table V.), up to about 1750, when we have the Wolf numbers. Apparently there was a definite change about this time. Instead of the 8·75-year term, which drops out or becomes very small (§ 28) we get a 9·96-year period (§ 27 and also § 9). But more clearly still is shown a 13-year period, which seems to fluctuate.

3. The fluctuation of the 13-year period is best shown by the long series from 1610, for apparently it existed from that date, though its coefficient was small (see Table VII.). It was much enhanced about 1728, and was thus more easily found in the later observations. The fluctuation takes place in 130 years (see Table VII.).

4. After removing this fluctuating term, we get a good value for the 11-year period as 67/6 years (§ 21), confirmed by the long series (§ 14). Further, the difference between consecutive 11-year cycles (which may be connected with the reversal of magnetic polarity at minimum) is now rendered systematic (§ 23), whereas on the supposition of an 11·5-year period (suggested by observations from 1880 for which we have the Greenwich measures) this alternation is much less clearly shown (§ 24).

5. These results represent a view of sunspot periodicity quite different from that worked out in *M.N.*, **74**, 82. The fluctuations were there ascribed to a single swarm of meteors with a variable orbit. The present treatment would correspond to a combination of several swarms. But both treatments suggest epochs of discontinuity and the examination of the early records from 1610 to 1750 has suggested a new one, 1734 (see § 12), which accords with those previously noticed in being near a return of the Leonids. No preference is at present suggested for this later analysis over the former. It was undertaken with the quite definite and different object of tracing some origin for the magnetic period, which has completely failed; and for the present the matter must be left there.

Micrometrical Measures of Double Stars. By Rev. T. E. Espin and W. Milburn.

In the following list the stars marked E were measured with the 24-inch Calver reflector, and M with the 17¼-inch reflector by the same maker.

Name.	1900. R.A. h m	Decl. ° ′	P. °	D. ″	Mags.	Nts.	Date 1925.	
Espin 114	0 5·4	+66 35	164·4	4·40	8·6 10·3	2	·965	M
Espin 113	5·6	66 42	129·9	6·70	8·5 9·8	2	·965	M
Σ 19	11·5	36 0	134·4	2·31	6·8 10·0	3	·022	E
Milburn 1	21·5	40 18	232·0	7·19		3	·589	E
Milburn 2	24·3	39 37	231·4	3·75	10·5 10·7	3	·021	E
Σ 41	29·7	38 37	189·7	15·58	8·5 9·6	2	·018	E
Milburn 3	41·7	38 28	23·0	7·76		3	·589	E
Milburn 4	42·9	38 51	97·0	6·17	9·5 11·0	3	·021	E
Milburn 5	55·2	39 57	340·7	6·72	9·6 10·0	2	·031	E
Doo——	1 7·1	35 4	217·2	1·52	9·2 9·3	4	·914	E
h——	12·0	65 35	14·2	7·20	9·5 10·0	3	·025	M
Σ 105	12·2	65 38	177·9	3·06		3	·025	M
Milburn 6	30·9	39 36	234·0	6·26	10·5 10·5	3	·334	E
Milburn 7	36·1	39 4	316·6	3·85	9·4 11·5	2	·032	E
Milburn 8	37·2	38 46	95·8	6·45	8·6 9·8	2	·058	E
Milburn 9	46·3	40 55	320·7	5·93	8·9 9·5	2	·095	E
Milburn 10	57·4	38 50	53·1	4·16		2	·515	E
Milburn 11	58·2	39 37	125·6	2·75	9·3 10·7	2	·519	E
Milburn 12	2 2·9	39 5	254·0	5·51	8·7 9·4	3	·374	E
Milburn 13	34·6	39 48	131·5	7·70	9·5 9·7	2	·946	E
Hu 604	42·1	35 55	196·7	2·20	9·0 10·5	4	·934	E
Milburn 14	3 0·4	39 29	6·9	5·04	10·3 10·7	2	·946	E
Milburn 15	0·7	39 44	228·0	2·93	10·3 11·7	2	·946	E
Σ 484	59·0	62 4	132·1	5·32	9·5 10·2	2	·993	M
Σ 485	59·1	62 4	255·7	5·82	6·0 12·0	2	·993	MAB
Holmes 3	59·3	62 4	214·5	5·44	9·5 10·0	2	·993	M
Σ 658	5 9·8	38 56	185·7	5·77	8·5 9·0	2	·119	E
Σ 739	33·4	66 30	245·7	2·22		2	·974	M
A 2115	6 12·4	39 37	178·8	1·62	9·2 9·4	1	·107	E
Espin 290	7 14·4	36 55	308·5	4·46	9·3 10·5	2	·223	E
β 901	15·4	36 57	7·0	11·06	5·0 11·5	3	·227	E
Espin 424	8 0·6	35 15	71·2	5·71	9·3 11·0	1	·222	E
Espin 300	9 27·6	35 36	143·0	1·94	9·3 9·4	3	·312	E
h 2525	10 12·6	37 0	75·2	16·47	11·0 11·2	2	·308	E
h 3326	19·1	36 28	176·3	19·21	10·2 11·5	3	·312	E

Name.	1900. R.A.	1900. Decl.	P.	D.	Mags.	Nts.	Date. 1925.	
	h m	° ′	°	″				
Milburn 66	10 40·0	36 29	179·7	4·48	9·5 10·3	2	·315	E
Espin 1737	41·1	36 10	281·7	2·10	9·3 9·8	2	·315	E
Espin 181	11 6·6	36 22	144·0	4·91	9·3 10·5	1	·320	E
Scheiner 1300	20 49·9	36 39	122·0	1·75	9·5 9·7	3	·859	E
h 3027	21 18·8	70 44	122·2	33·98	9·5 9·6	2	·877	M
h 3035	24·9	72 7	130·2	23·83	9·5 9·8	3	·763	M AB
			203·6	24·44	C=12·0	2	·771	M AC
Hn 45	26·2	34 37	196·9	1·47	9·2 9·3	6	·914	E
h 3051	35·3	72 20	177·2	13·17	9·8 10·0	2	·877	M
Ho 164	36·8	34 37	64·3	3·29	8·5 8·6	2	·912	E
Ho 605	39·8	34 26	341·1	1·80	10·0 10·3	3	·882	E
Ho 466	43·1	34 26	141·6	2·01	9·0 11·2	3	·938	E
β 1214	52·2	33 51	206·4	1·71	9·0 9·3	3	·956	E AB
			243·6	4·90	9·3 10·2	3	·938	E CD
h 3081	53·4	72 39	318·2	32·87	9·3 10·0	3	·886	M
Espin 386	59·7	34 4	34·7	6·20	8·8 9·7	2	·877	E
h 3096	22 4·7	70 29	339·7	8·35	9·5 10·1	3	·872	M
Espin 214	10·9	34 17	165·2	3·41	9·2 11·5	1	·977	E
Espin 215	11·0	34 24	140·8	8·08	8·5 11·5	1	·977	E
h 973	49·4	34 55	260·1	10·13	11·5 11·7	2	·991	E
h 1851	23 5·0	69 34	343·0	3·10	8·5 9·0	2	·931	M
β 386	22·0	70 8	313·3	20·46	6·3 11·0	2	·993	M
h 986	22·2	34 47	286·7	9 59	9·5 12·0	2	·991	E
Ho 201	32·2	34 6	342·7	3·58	9·0 9·5	2	·892	E

Notes.

Σ 658. Two *comites* 12 mag., P. 43°·3, D. 23″·5, and P. 332°·8, D. 25″·0.
Scheiner 1300. Smaller star was certainly *p.* on all three nights.
Espin 386. B.D.+33°·4417, not 33°·4427. Scheiner 1553.

New Double Stars.

The following double stars have been found and measured with the 24-inch Calver reflector.

No. E.	B.D.	1900. R.A.	1900. Decl.	P.	D.	Mags.	Nts.	Date 1925.
	°	h m	° ′	°	″			
2138		0 19·6	+34 31	49·1	4·20	9·6 11·0	3	·874
2139	+34·93	33·2	34 43	68·8	4·89	9·5 14·0	2	·866
2140	+33·103	40·0	34 1	229·6	2·22	9·4 11·5	2	·902
2141	+35·212	1 3·8	35 40	271·4	4·20	9·3 9·5	2	·014
2142		27·9	34 55	246·8	4·23	9·4 9·7	2	·858
2143		45·0	34 25	182·9	2·48	9·8 10·7	2	·880
2144	+34·339	50·8	34 36	142·6	6·47	9·2 9·3	2	·873

No. E.	B.D.	1900. R.A. h m	1900. Decl. ° ′	P. °	D. ″	Mags.	Nts.	Date 1925.
2145	+36·522	2 30·7	36 13	26·2	3·81	9·3 9·5	3	·620
2146		50·1	36 52	93·7	3·31	9·8 10·5	3	·103 AB
				66·7	12·28	C=10·0	3	·103 AC
2147	+37·764	3 18·8	38 4	55·6	2·83	9·5 10·8	3	·956
2148		4 6·1	36 58	180·1	1·95	10·5 11·0	3	·972
2149	+38·911	27·9	38 11	221·0	5·76	8·8 9·2	2	·102
2150	+39·1131	52·6	39 11	35·2	2·12	9·5 11·0	2	·102
2151	+38·1137	5 16·5	38 57	111·1	5·68	8·8 11·7	3	·171
2152	+38·1219	29·6	38 59	321·2	3·10	9·5 11·0	2	·119
2153	+38·1336	50·4	38 50	76·6	3·66	8·5 10·0	2	·119
2154	+39·1555	6 7·8	39 21	359·6	2·67	9·5 12·0	2	·157
2155	+38·1543	30·1	38 55	54·2	4·56	9·0 13·0	3	·161
2156	+38·1690	7 0·7	38 36	222·7	5 54	9·1 9·3	2	·119
2157	+37·1743	29·4	37 5	166·1	2·40	9·5 10·5	3	·228
2158	+37·1757	35·3	37 9	332·1	2·96	9·3 11·0	2	·119
2159	+36·1757	8 4·4	36 13	54·2	7·95	8·5 9·2	3	·169
2160	+36·1818	19·7	36 28	342·5	3·71	9·1 9·2	2	·119
2161		25·4	35 44	5·5	3·70	10·0 10·5	2	·266
2162		9 12·7	35 33	102·2	3·96	10·5 12·0	3	·295
2163		29·8	36 0	94·6	4·06	10·7 11·7	2	·223
2164	+35·2170	10 34·3	35 36	87·2	1·89	9·3 9·5	2	·266
2165	+36·2167	11 8·8	36 26	87·6	1·85	9·3 9·5	2	·315
2166		12 30·4	37 7	354·4	4·31	10·7 11·0	2	·333
2167	+38·2353	34·9	37 46	269·9	5·95	9·5 9·7	2	·304
2168	+36·2931	17 40·3	36 58	287·9	5·74	9·5 10·1	3	·614
2169	+36·2951	46·1	36 51	336·2	8·47	9·3 9·6	2	·620
2170	+35·3163	18 5·8	35 46	261·5	7·31	9·3 13·0	3	·640
2171		7·9	35 57	70·3	3·79	10·2 10·7	2	·638
2172		19·2	35 19	39·1	3·08	10·5 11·5	3	·667
2173	+36·4130	23·2	36 6	300·3	5·56	7·8 12·0	2	·605
2174	+36·3454	19 10·0	36 12	157·7	5·41	9·3 9·4	2	·636
2175		10·6	36 36	156·2	2·32	10·0 12·0	4	·639
2176	+36·3476	12·6	36 10	189·8	5·43	9·4 12·0	4	·671
2177	+36·3517	17·9	36 23	180·8	2·48	9·5 10·5	3	·695 AB
				153·6	11·87	C=13·7	3	·782 AC
				262·7	16·63	D=13·0	3	·731 AD
2178	+36·3546	21·3	36 41	130·7	2·50	9·5 9·7	2	·605
2179	+36·3554	22·1	36 46	132·0	6·29	9·0 11·5	2	·605
2180	+36·3629	32·9	36 24	120·1	6·05	8·7 10·7	3	·685
2181	+36·3660	37·1	36 24	121·7	2·54	9·3 10·0	2	·760

No. E.	B.D.	1900. R.A.	1900. Decl.	P.	D.	Mags.	Nts.	Date 1925.
	°	h m	° ′	°	″			
2182	+35·3803	19 43·9	35 42	42·2	1·98	9·3 9·4	3	·843
2183		51·6	36 0	96·5	2·54	10·0 11·5	2	·835
2184	+36·3802	55·6	36 16	84·7	4·45	9·2 13·0	2	·758
2185	+36·3890	20 4·6	36 56	240·8	1·99	9·4 9·5	2	·856
2186	+37·3799	6·4	37 11	186·1	3·78	9·5 13·0	4	·683
2187	+37·3810	7·5	37 8	342·0	3·16	9·5 12·5	3	·704
2188	+36·3936	9·3	36 11	67·2	1·81	9·3 11·2	4	·874
2189	+36·3969	11·8	36 55	105·3	5·87	9·3 12·5	3	·846
2190	+36·4037	18·9	36 32	167·5	1·51	9·5 9·6	4	·870
2191	+35·4095	19·7	35 59	179·1	2·16	8·3 11·0	3	·886
2192		21·9	36 49	83·7	2·64	9·7 12·0	3	·738
2193	+35·4126	23·9	35 45	337·0	4·01	9·3 10·5	4	·902 AB
				230·2	6·60	C=14·0	2	·898 AC
				33·2	13·24	D=11·0	4	·902 AD
2194	+36·4169	35·8	36 49	296·6	6·29	9·5 12·5	2	·794
2195	+35·4235	39·6	35 47	92·2	1·77	10·1 11·5	2	·883 BC
				362·7	22·93	A=8·7	3	·886 AB
2196	+35·4260	42·6	35 28	65·8	6·47	9·0 10·5	2	·920
2197	+36·4373	57·5	37 3	82·2	3·83	9·5 12·0	2	·771
2198		59·3	35 27	28·4	2·87	10·0 10·2	2	·931
2199	+35·4450	21 11·2	35 52	93·8	4·12	8·7 12·5	3	·886
2200	+34·4544	46·0	34 23	173·3	1·22	9·7 10·7	3	·860 BC
	h 1699			59·2	13·45	A=9·3	2	·861 AB
2201	+33·4491	22 18·4	33 32	278·6	2·99	9·3 13·0	2	·946
2202	+33·4536	29·8	33 53	271·4	2·33	9·5 12·0	4	·912
2203		39·7	34 11	90·7	2·14	9·5 9·6	3	·799
2204	+33·4620	54·0	33 50	163·0	2·31	9·5 9·6	3	·927
2205	+33·4639	59·4	34 12	78·0	4·70	9·5 12·0	2	·813
2206		23 25·2	33 4	357·2	2·06	10·5 10·9	3	·956
2207	+33·4743	30·5	34 8	243·3	6·91	9·0 10·5	2	·860
2208	+34·4965	32·0	34 23	81·9	2·75	9·4 9·6	3	·829
2209	+33·4823	58·2	33 23	240·3	4·41	8·9 12·0	2	·965

Notes.

2138. S of two stars.
2162. In field with B.D.+35°·1974 at P. 67°·3, D. 60″.
2171. S of three stars. At P. 214°·6 from B.D.+35°·3175.
2177. C added by Mr. Milburn, distance uncertain.
2192. In field at P. 224°·3 with B.D.+36°·4065.
2193. AD is Scheiner 1128.
2200. *h* for AB gives P. 70°·0 and, under "Remarks" notes, "Points back to *h* 1697." This would be exactly true with P. 70°·0. It does so now no longer.

Stars Missing or Faint.

	h	m	s	°	′			
B.D. +34°268	1	24	29·3	+34	32·8.	9·5	1923 Nov. 9.	Not seen.
+35·574	2	42	2·8	35	54·2.	9·5	Dec. 4.	Not seen.
+37·790	3	24	52·1	37	22·4.	9·5	Dec. 10.	Not seen.
+38·920	4	28	23·1	38	45·8.	9·5	1924 Dec. 14.	Not seen.
+38·925	4	32	10·9	38	4·4.	9·5	1925 Feb. 6, Feb. 9.	Not seen.
+39·1459	5	47	9·0	39	21·0.	9·5	Mar. 9.	Not seen.
+35·2134	10	14	53·9	35	20·1.	9·5	Apr. 23.	Not seen.
+38·2393	12	54	18·4	38	20·3.	9·5	Apr. 24.	13 mag.
+36·2935	17	40	46·8	36	56·7.	9·5	Sept. 4.	Not seen.
+33·4726	23	24	58·0	33	4·0.	9·5	Dec. 10.	No star here.

New Double Stars.

The following double stars have been found and measured with the 17¼-inch Calver reflector.

No. M.	B.D.	1900. R.A.	1900. Decl.	P.	D.	Mags.	Nts.	Date 1925.
	°	h m	° ′	°	″			
377	+66·34	0 26·2	+66 42	93·4	2·79	9·5 12·0	2	·965
378	+61·140	33·6	61 25	352·5	6·46	9·4 11·7	3	·876
379		34·0	61 24	174·8	5·49	9·5 9·7	3	·876 AB
				205·5	5·70	C=9·6	3	·876 BC
380		34·2	67 0	218·0	2·14	9·7 10·8	2	·993
		36·2	61 31	75·8	7·51	9·5 11·5	2	·865
381		37·0	61 23	211·8	2·50	9·5 9·7	3	·859
382		1 21·0	62 50	107·6	4·17	9·5 10·0	2	·008 AB
				94·0	54·63	C=11·0	1	·005 AC
				28·8	72·63	D=9·6	1	·005 AD
383	+62·279	30·4	63 13	161·7	5·59	9·5 12·0	2	·858 AB
				64·4	32·24	C=12·0	3	·861 AC
				169·5	32·34	D=9·5	3	·861 AD
384		34·8	62 41	218·4	2·24	9·6 11·0	4	·906
385		40·0	62 54	239·8	4·06	12·0 12·7	3	·914
386		56·7	62 19	69·0	2·76	10·0 11·0	3	·894
387		2 0·6	62 6	253·8	2·45	10·0 10·3	2	·865
388		28·1	63 2	99·5	2·73	9·8 10·7	3	·861
389		29·7	62 32	123·5	3·78	11·0 12·3	3	·900
390	+62·491	48·4	62 58	29·9	4·55	9·5 11·0	2	·965
391		52·0	62 20	160·4	4·40	10·0 10·1	2	·965
392	+65·313	52·5	66 10	25·0	6·89	9·0 9·9	2	·511
393	+65·367	3 39·8	65 20	310·2	3·14	9·4 9·5	2	·931

No. M.	B.D.	1900. R.A.	1900. Decl.	P.	D.	Mags.	Nts.	Date 1925.
	°	h m	° ′	°	″			
394		3 41·0	65 50	317·3	5·82	9·4 9·4	3	·914 AB
				191·1	12·52	C=9·5	2	·902 AC
395		48·7	65 21	28·2	7·01	9·6 12·0	2	·114
396		4 4·8	65 55	96·3	4·40	10·5 10·6	3	·094
397	+65·443	48·9	65 2	287·1	4·37	9·6 9·6	3	·682
	+66·442	6 16·4	66 0	13·2	9·84	9·5 10·2	2	·157
398	+66·445	20·8	66 41	155·8	4·82	9·0 11·5	3	·112
399	+67·457	41·5	67 34	258±	7±	9·0 11·0	1	·192
	+67·460	44·0	66 58	166·7	9·17	9·5 10·0	3	·112
400		51·7	66 25	238·3	2·45	9·5 9·8	2	·187
401	+66·477	51·7	66 45	140·6	3·90	9·1 9·6	2	·187
402	+67·485	7 13·3	67 25	167·5	5·63	8·0 10·0	3	·140
403		40·7	66 10	288·7	4·42	9·8 11·0	3	·140
		8 11·1	66 8	113·5	8·19	9·8 11·3	2	·119
	+69·475	26·1	69 8	306·1	9·77	9·3 10·5	2	·238
404		58·6	68 22	159·6	3·48	9·8 11·5	2	·237
	+66·595	59·4	66 46	329·4	9·84	9·5 10·0	2	·157
405	+67·611	9 35·0	67 36	106·9	6·33	9·0 11·5	4	·188
406	+67·654	10 20·9	67 38	42·6	3·61	9·4 10·5	5	·261
407		56·9	65 34	79·1	4·69	10·0 10·7	3	·287
408	+65·820	11 4·6	65 15	344·9	8·16	9·0 10·0	2	·315
	+64·864	39·5	64 12	55·9	8·54	9·5 11·5	2	·276
409	+64·870	42·5	63 58	161·3	2·10	9·5 10·2	3	·287
410		12 10·4	64 32	38·5	5·82	11·5 11·7	1	·309
411		38·5	62 37	186·2	4·67	9·5 10·0	2	·315
412		13 4·5	63 10	269·9	2·68	9·5 11·0	1	·356
413		18 19·9	67 40	149·8	2·70	10·1 10·6	4	·610
414	+67·1105	58·1	67 7	148·2	2·49	8·7 9·9	2	·613
		59·8	66 11	121·1	6·58	11·5 13·0	2	·620
		19 40·8	69 32	92·7	9·00	9·5 10·7	2	·658
415	+69·1067	43·5	69 15	348·5	8·62	8·5 9·3	2	·663
		49·8	69 42	337·7	6·40	11·0 12·0	3	·710
416	+71·1006	20 11·1	71 14	164·1	6·28	9·5 11·0	2	·794
417		14·7	36 39	92·3	3·09	10·7 11·2	2	·794 E
418		15·8	70 32	48·4	3·75	10·5 11·2	3	·615
419		19·5	71 5	133·8	2·26	9·5 9·6	2	·643
420		37·9	70 58	269·8	3·21	9·7 11·5	2	·620
421		45·9	71 57	253·6	1·89	10·0 10·6	3	·865
422	+70·1147	49·1	71 9	194·1	7·71	9·5 12·0	3	·610
423		59·0	72 0	245·0	5·34	11·0 11·5	3	·706

No. M.	B.D.	1900. R.A.	1900. Decl.	P.	D.	Mags.	Nts.	Date 1925.
	°	h m	° ′	°	″			
424		21 11·5	36 55	64·7	3·68	9·3 10·7	2	·758
425		30·3	73 15	333·6	2·10	9·5 9·6	2	·888
426	+72·1000	47·0	72 35	116·9	3·06	9·5 13·2	3	·886
427		55·0	72 52	154·9	2·91	9·5 10·2	2	·912
428		56·4	71 44	77·3	5·55	9·5 11·5	3	·863
429	+71·1102	57·6	71 56	228·7	2·91	8·9 9·2	2	·861
430		22 2·4	67 53	295·6	5·27	11·0 11·2	4	·776
431		2·8	68 35	139·6	4·13	9·7 9·8	2	·931
	+67·1414	8·0	68 9	167·2	12·68	8·7 10·0	2	·871
432		17·4	68 5	177·5	3·37	11·5 12·2	2	·877
433		47·7	69 26	214·0	2·83	9·5 9·7	2	·931
434	+55·2836	49·2	55 43	222·7	6·62	9·5 10·5	3	·829
435	+68·1349	58·0	68 40	48·0	6·51	8·5 9·4	2	·880
436	+71·1185	23 4·8	71 20	347·2	3·98	9·3 12·0	2	·898
437	+69·1312	6·4	69 53	140·2	4·59	9·5 9·7	2	·920
438	+69·1316	8·8	70 5	52·0	2·91	9·5 12·3	2	·911
439		39·8	70 18	230·8	3·71	10·1 11·8	3	·926
440		43·7	69 57	84·3	3·21	9·6 11·0	3	·909

Note.

384. This star precedes the place of J878 by $1^m\ 46^s{\cdot}1$, and the measures differ widely.

On the Cause of Anomalous Determination of Time.

By M. R. Madwar, B.Sc.

(*Communicated by Professor R. A. Sampson, F.R.S.*)

In *M.N.*, **82**, 221, Professor R. A. Sampson showed that the irregularities in the trace of the clock correction could neither be ascribed to the chronographic system nor to a diurnal fluctuation in the clock rate, but are, as a rule, due to our adopted instrumental constants. These are the well-known quantities c, m, n as used in Mayer's formula.

The procedure at the Royal Observatory, Edinburgh, included, besides these, the determination of the quantities A and $[\alpha]$; where A denotes the azimuth of the transit circle east of the South collimator and $[\alpha]$ the azimuth of the North collimator.

It was also shown in *M.N.*, **85**, 560, that the elimination of the level error and the introduction of A in the time reduction was responsible for the disappearance of a great number of the above-mentioned irregularities or erratics in the trace of the errors of the clock (Shortt No. 4) during 1924. With a view to explore this method further the errors of Riefler 258, from 1915 January to 1923 December, have

been computed by both methods of reduction. The charts attached show the traces of the observed (full line) and adopted (dotted line) clock errors on a time basis. It will be readily seen that a great number of erratics resulting from the ordinary method of reduction are eliminated, some are only reduced, and the reduction with A and n introduces elsewhere erratics smaller in number and in variation.

The mean erratic during the nine selected years is reduced from $\pm$·03 sec. to $\pm$·02 sec. This in itself is a large reduction, for the latter includes all errors due to personality and those of an accidental nature pertaining to the instrumental constants, the chronographic system, and all errors due to a lateral component in atmospheric refraction, whether inside the telescope, in the transit-circle house, or outside; and finally, those due to errors in the R.A. of stars. But apart from this reduction, the aim was to show that certain of the large erratics introduced by the ordinary method of reduction have been eliminated when the reduction is carried out with A and n. The azimuth A of the transit circle has been derived from an arbitrary zero (·397), and thus the clock errors are not absolute; however, we are more concerned with the variation of the clock errors than their absolute values, and the fixing of the datum line has no effect on the outcome.

The difference between the adopted clock error when the reduction is made with l and the adopted error when the reduction is made with A, n shows a seasonal variation which can only be due to either or to both methods of reduction. The following table shows the monthly mean of the above-mentioned difference for every year and for the nine selected years :—

	1915.	1916.	1917.	1918.	1919.	1920.	1921.	1922.	1923.	Mean.
	″	″	″	″	″	″	″	″	″	″
Jan.	·07	..	·05	·17	·09	·08	·05	..	·05	·080
Feb.	·09	..	..	..	..	·16	·00	..	·04	·073
Mar.	·11	..	·03	..	·10	·11	·04	·03	·07	·070
Apr.	·08	·05	·09	·01	·03	·07	·02	·02	·09	·051
May	·09	·12	·03	·03	·07	·11	·03	·06	·16	·078
June	·17	·14	·10	·03	·13	·13	·04	·11	..	·106
July	·12	·10	·14	·02	·01	·03	·05	·07	..	·068
Aug.	·16	·08	·07	·02	·11	·04	·06	·05	·15	·082
Sept.	·21	·12	·09	·04	..	·10	·11	·10	·16	·117
Oct.	·27	·08	·04	—·02	·15	·07	·11	·10	·20	·110
Nov.	·29	·07	..	·11	·16	·06	·13	·10	·24	·129
Dec.	·16	·03	·14	·13	·12	·08	·02	·06	·24	·120

The maxima and minima do not occur at the same time, but undergo a slight shift from year to year. For the nine years selected the lowest value occurs in April and the highest value in November, the difference between them being ″·08. This discrepancy may be either due to the eliminated level error l or to the introduced quantity A; for both these quantities show seasonal variations. However, the South collimator appears to be fairly steady (*M.N.*, **75**, 74), whereas the level

error varies sensibly from night to night and also shows seasonal variation, following the temperature of the eastern pier of the transit circle. But since the method of time reduction with A and n has given a decided improvement in the trace of clock errors, it appears a probable presumption that the level error introduces a seasonal variation in the adopted clock error. Also the comparison of wireless signals from Annapolis with the mean of six observatories shows a decided seasonal error in the time reduction of Washington (*M.N.*, **82**, 219); and the comparison between Edinburgh and Annapolis shows an improvement in this seasonal error when the reduction is carried with A, n instead of l.

The level error is determined by means of a mercury bath and a Bohnenberger eyepiece, the movable wire of the telescope is made to coincide with the reflected image of the middle wire, and the reading of the micrometer gives a measure of the level error. This reading is taken at the beginning and at the end of each observation; but as the mercury trough has to be removed during the observing period, the two readings form two independent determinations of the level error; they differ very slightly, and this rules out of court any accidental errors in the determination of l, but leaves untouched any errors that have entered equally and with the same sign into both determinations.

On the assumption that the earth's crust is insensible to the tidal actions of sun and moon and to the effect of diurnal variation of temperature, we may form an upper limit of the errors contributed by these causes, which are tacitly and wrongly debited to the instrumental level l, and consequently reappear in the clock correction.

The moon tide-generating potential at any point on the earth's surface is the spherical harmonic

$$\frac{GM}{R^3}r^2\frac{(3\cos^2 z-1)}{2} \qquad \frac{GM}{R^3}p^2\frac{(3\cos^2 z-1)}{2}$$

where M is the mass of the moon.
R, distance to the earth's centre.
r, radius of earth at the point considered.
G, gravitation constant.
z, angle between the line joining the earth and moon and the earth's radius.

The horizontal and the vertical component of the tide-generating force is:

$$F_h=-\frac{3}{2}\frac{GM}{R^3}r\sin 2z.$$

$$F_v=\frac{GM}{R^3}r(3\cos^2 z-1).$$

The first force produces a deviation in the direction of gravity, and the second alters its intensity; this deviation is

$$\frac{3}{2}\left(\frac{M}{M'}\right)\left(\frac{r}{R}\right)^3\sin 2z=''\cdot 018\sin 2z.$$

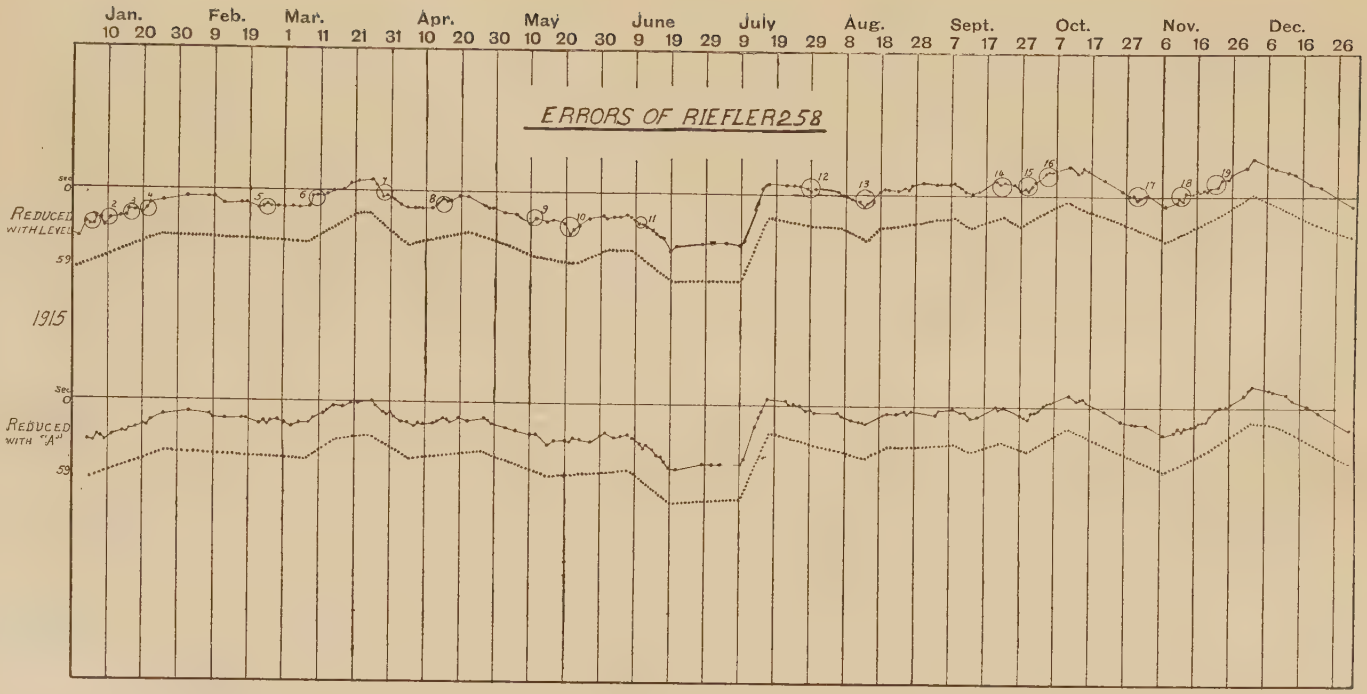
The sun's effect, owing to its remote distance and greater mass, produces a deviation $=''\cdot008 \sin 2z$, and will be additive when the moon is in opposition or conjunction, and subtractive when it is in quadratures.

But the rising tide may produce on a plumb line near the coast a deviation equal to, if not greater than, that due to direct action of the sun and moon. An approximate calculation shows that this cannot be greater than $''\cdot025$. This would make a total maximum deviation of $''\cdot050$, which would be responsible for an erratic whose greatest value is $^s\cdot006$ only; a very small and almost negligible quantity.

The diurnal variation in temperature may likewise produce different effects on the mercury bath and on the earth's crust, and this difference produces an error in the estimation of the instrumental level l; but the estimation of this error does not admit of a simple treatment, as in the previous case; only we may mention that the experiments of O. Hecker with the horizontal pendulum in 1907 * show that the mean diurnal variation of the direction of gravity is very small also, and does not produce an erratic feature in the time determination more than $^s\cdot004$. It appears from this discussion that there is no theoretical error in the value of l, as determined by the process adopted, or that these theoretical errors are so very small that they could hardly account for the erratics; and yet a great number of these are smoothed away when the reduction is carried out without the level error.

In conclusion I wish to express my deep gratitude to Professor R. A. Sampson for his kind interest and advice.

* O. Hecker, "Beobachtungen an Horizontalpendeln," *Veröff. d. Konigl. preuzz. geodät. Inst.*, 1907; Lallemand, *Bulletin Astronomique*, **33**, 369, 1911.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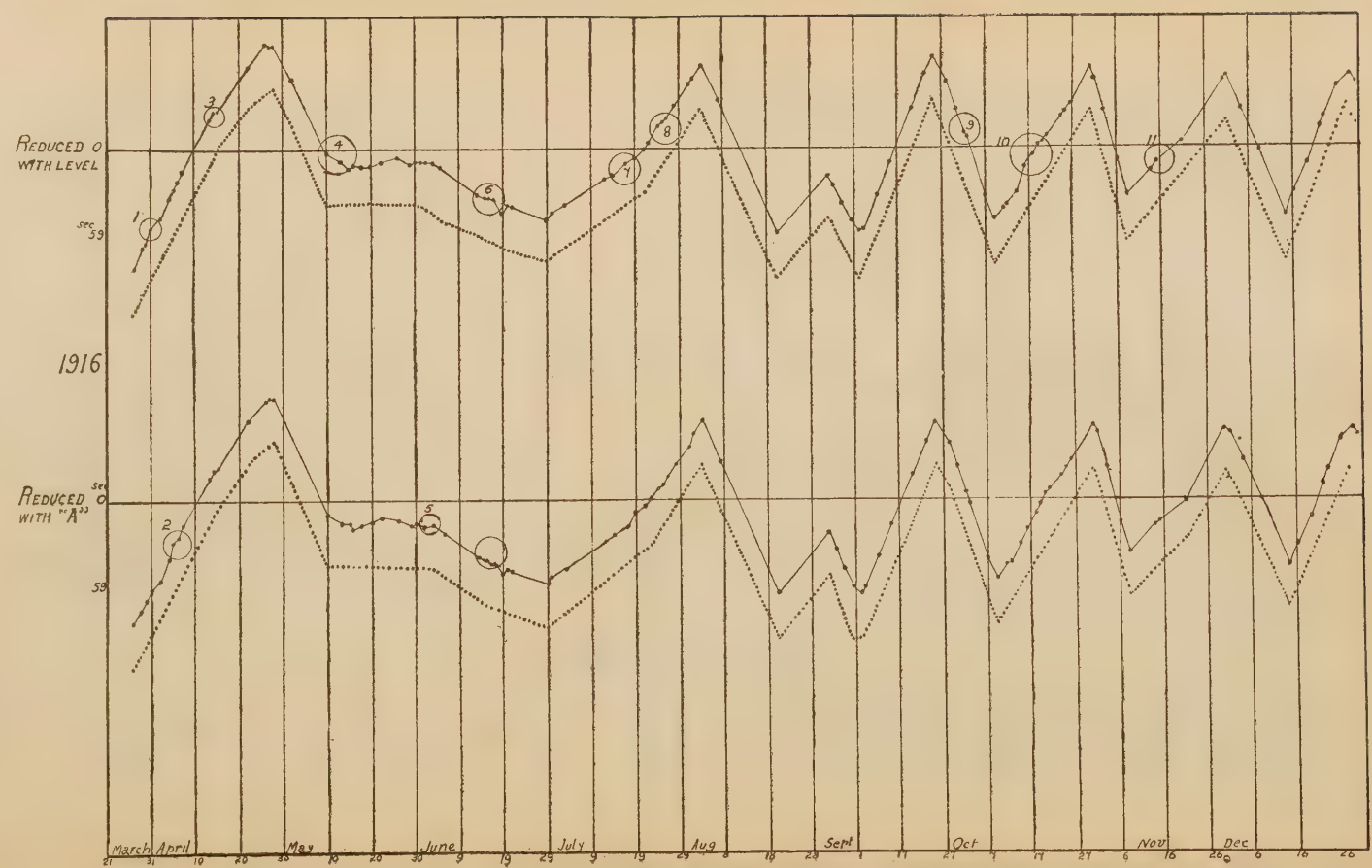
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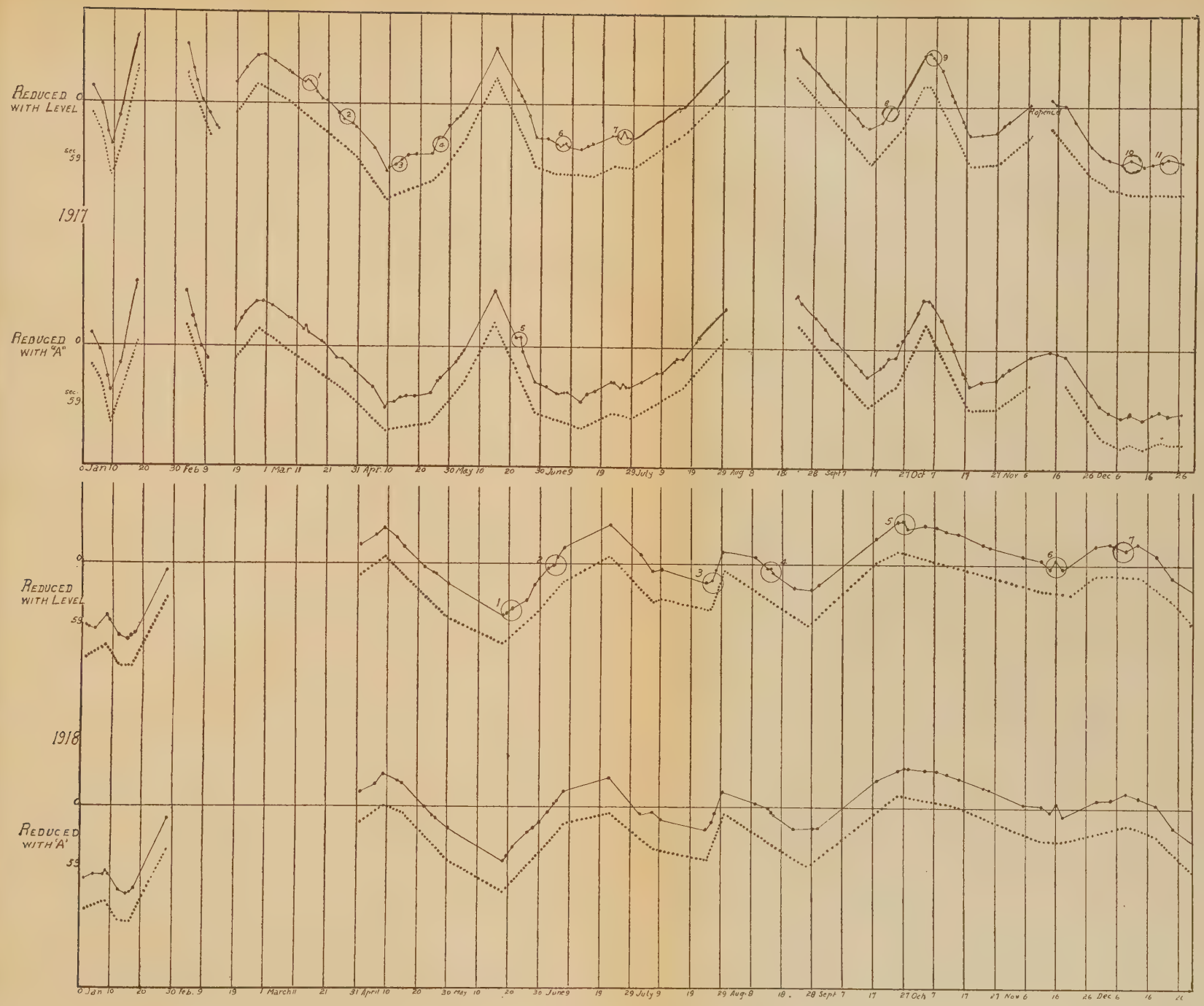
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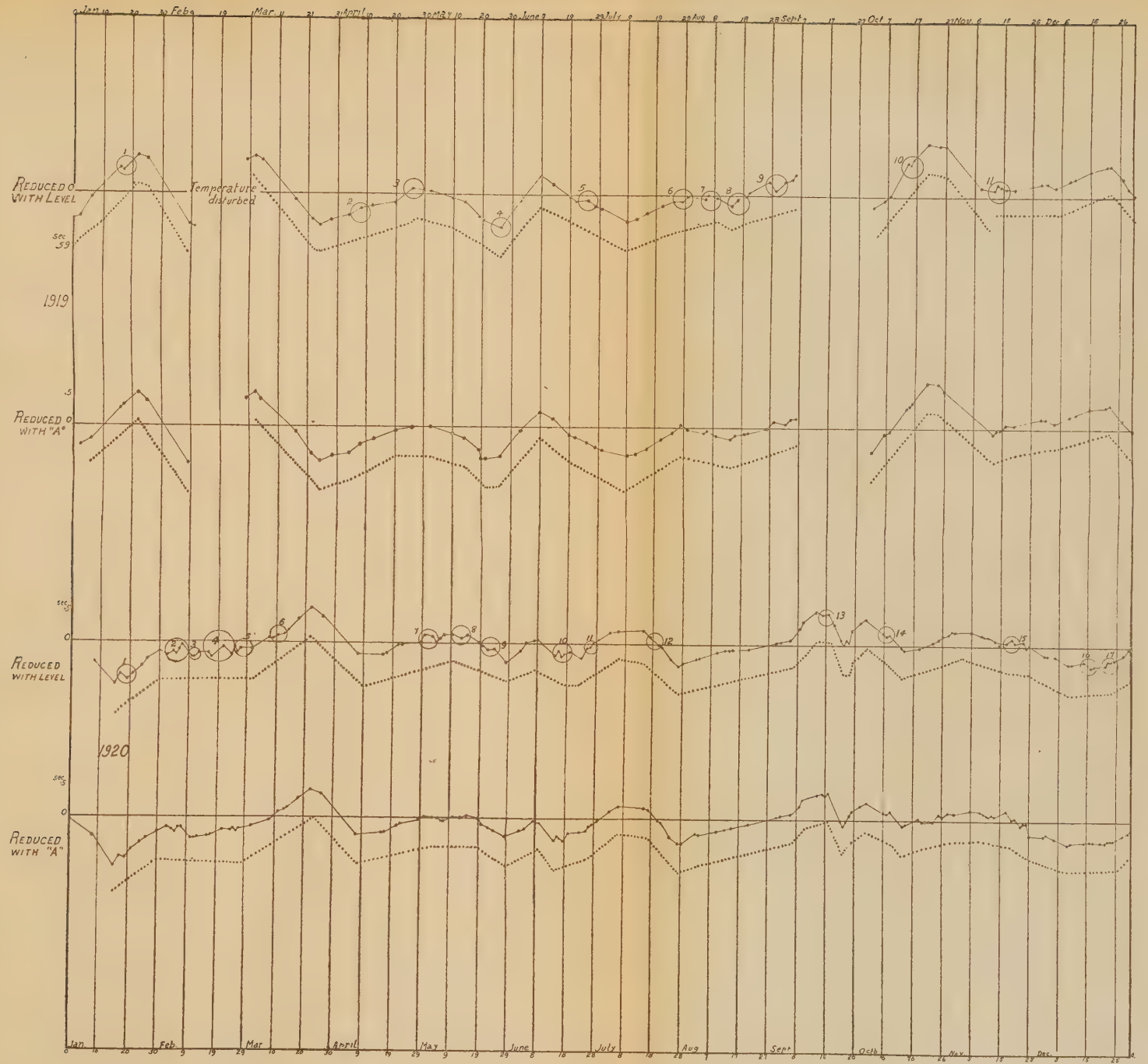
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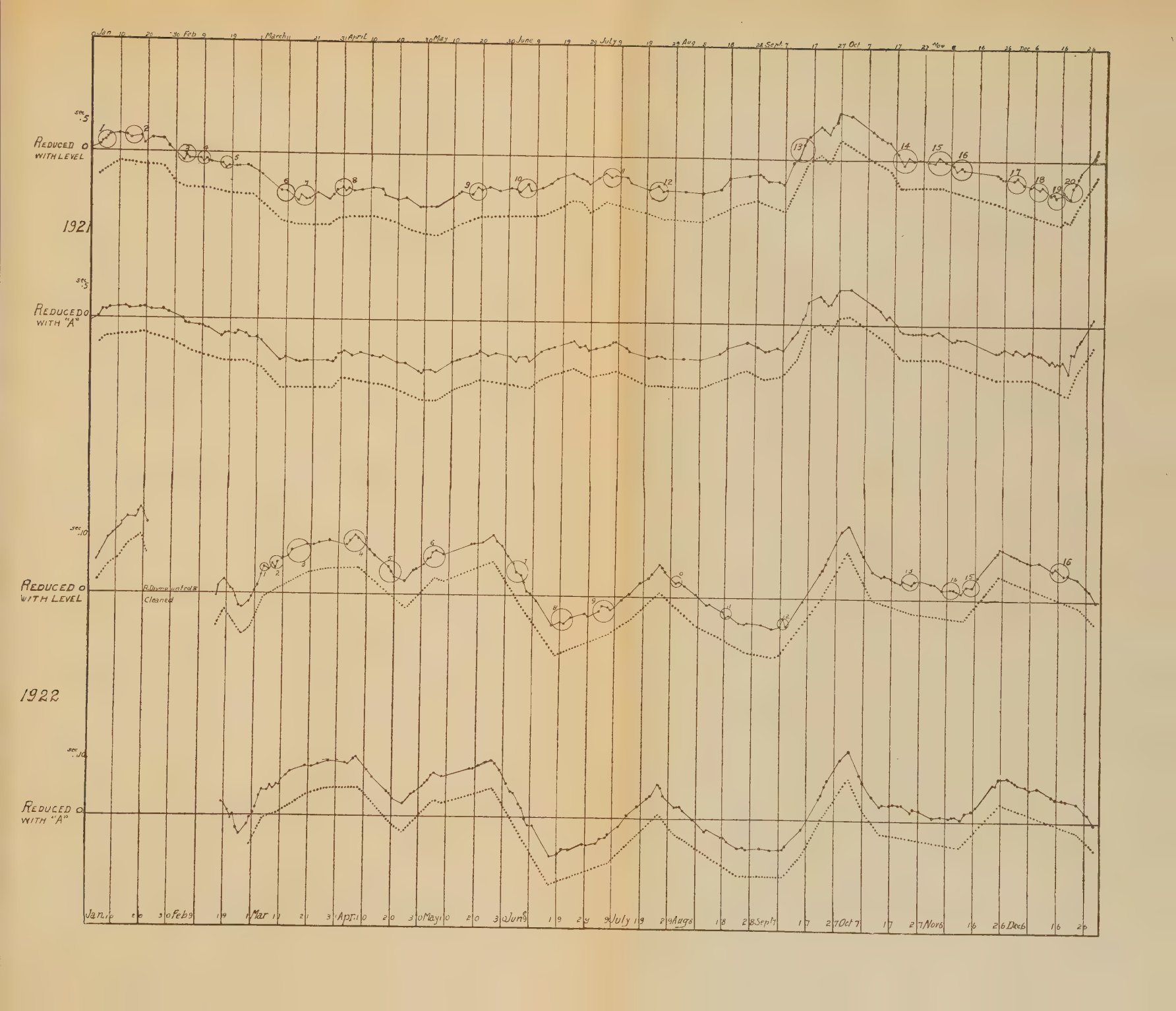
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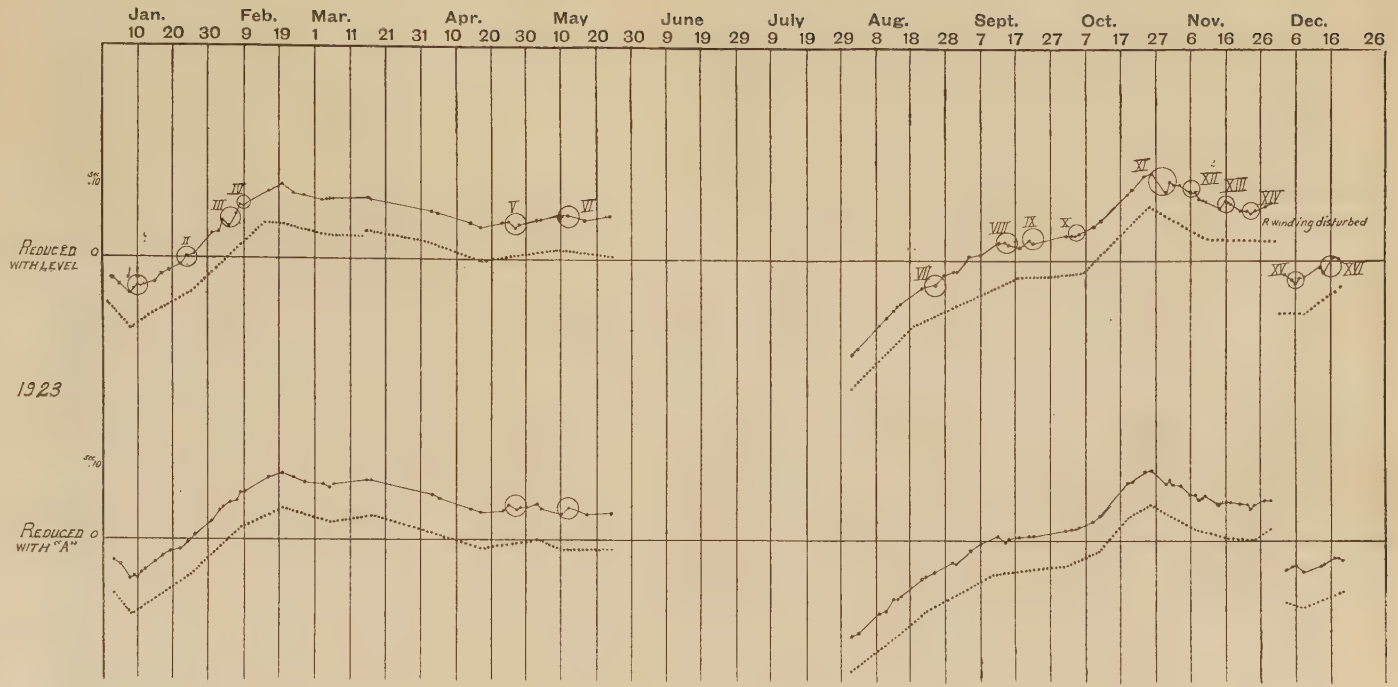
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First Note on W. Herschel's Observations of Extended Nebulous Fields. By J. G. Hagen, S.J., Assoc.R.A.S.

Sir W. Herschel published a list of fifty-two fields of "extensive diffused nebulosity," giving their positions for 1800 (*Phil. Trans.*, 1811). His table was reprinted by Barnard in *Knowledge*, **15** (1892), 14, and again by Isaac Roberts in *M.N.*, **63** (1903), 26, where the positions are reduced to the equinox of 1900. Twenty years later the present writer published Herschel's list anew in the *Atti* of the Academy Nuovi Lincei, **76** (1923), 200, with positions for 1925.

1. To facilitate identification of these "extensive nebulosities" in the heavens a few charts were made with our astrographic refractor and the reticule impressed upon them. For the guiding of the photographs the brightest star was chosen in each case that could be found within half a degree of Herschel's position. A ten minutes' exposure was sufficient, although a longer one would have been more useful. The astrographic plates cover, as is known, exactly two degrees in each co-ordinate, and the squares of the reticule are five minutes on each side.

The centre of the charts corresponds to Herschel's positions, and the place of the guiding star is given in the following table by its $\Delta\alpha$ and $\Delta\delta$, in the sense (*—centre). When passing from the star to the middle of the field, the signs are to be reversed. The $\Delta\alpha$ were transformed into arc ($1^m = 15'\cos\delta$) for convenience in reading the reticule. The fields in question are Herschel's Nos. 1, 2, 3, 4.

TABLE I.

H.'s No.	Centre (1925).	Guiding Star (1925).			Mag.	$\Delta\alpha$.	$\Delta\delta$.
	h m ° ′			h m ° ′		′	′
1	0 11·4+ 9 34	Leipzig II.	61	0 12·9+ 9 50	6·8	+22	+16
2	0 18·9+ 4 7	Albany	56	16·5+ 4 22	8·3	−36	+15
3	0 23·7+29 17	Cambridge E.	267	26·2+29 20	6·0	+34	+ 3
4	0 26·9+ 4 7	Albany	88	26·3+ 4 27	7·0	− 9	+20

2. Herschel's description of the nebulosities is here tabulated from the *Scient. Papers*, **2**, 462. The main item is their extension "in the Parallel" and "in the Meridian," although he designates them as "uncertain," and especially those in Decl. as "unascertained." It could not be otherwise, as we shall see below.

It seemed preferable to divide Herschel's numbers by 2, counting the extensions from the middle of the chart to the margin. The size of the nebulæ in square degrees which Herschel computed is less important to the observer and was intended for the discussion that Herschel makes on p. 464.

The galactic latitude is added to the table with reference to which Herschel says, p. 465: "When we pursue it [the nebula in Orion] in its extensive course it assumes precisely the same appearance as the beforementioned diffused nebulosities." By "extensive course" he means the

TABLE II.

H.'s No.	Parallel.	Meridian.	Description.	Gal. Lat.
1	52′	57′	Much affected with neb.	−50°
2	90	77	Much affected	−58
3	20	80	Affected	−32
4	45	77	Much affected	−57

protraction of the nebula from where it is illuminated by stars into the dark fields that extend all round for several degrees, especially in a *sf* direction. It shows that Herschel saw the same kind of dark nebulæ in high galactic latitudes which are known to exist near the galactic plane.

3. Our visual refractor has a linear aperture of 16 inches, and the angular aperture 1 : 15 ; the eyepiece magnifies 122 diameters. The scale by which we estimate the density of dark nebulæ consists of five grades, I. denoting the thinnest veils and V. the greatest density (see our *Miscell. Astron.*, pp. 42 and 222).

Although the field of view is only 25′·4, the location of the estimates is very easy by means of the stars and of the reticule on the chart. The result of the observations, all confirmed by Dr. Becker, may be described for each chart as follows :

Herschel No. 1.—The centre of the field has the density V. to an extent of 15′ in directions *pnf*, but to 30′ southward. The rest of the field is IV.–V. throughout, and nowhere less than that. In the corners *sp*, *np*, and *sf* of the chart the density is again V.

No. 2.—Centre of the field=IV.–V., the rest=IV., but the *sf* corner=V. On the parallel of the guiding star and 67′ (=$4^{m}\cdot5$) following there is a clearer gap=III. of about 12′ in diameter and lined with stars from 10 to 12 mag.

No. 3.—This field is remarkable for its scarcity of stars, only two as bright as 9th mag. being found in a circle of 30′ around the centre. The density of the dark clouds in this circle is V., but the general density of the field is IV., except a few spots of V. or of III.–IV.

No. 4.—Centre of chart=IV., extending 25′ all round, but surrounded by densities IV.–V., nowhere reaching density V. The field is a continuation of field No. 2, both centres lying on the same parallel and two degrees apart. The two charts are contiguous, and the nebulæ on the common margin have densities IV. and IV.–V. Here we have a continuous field of dark clouds extending more than four degrees in R.A.

4. In conclusion, the following general statements can be made :—

(*a*) The four nebulous fields extend far beyond our charts, both by Herschel's estimates in Decl. (see Table II. above),

and by our observations. No limit could be found by sweeping.

(*b*) In no place of the charts is there any trace of an extended luminous nebula, a fact which may explain why the corresponding plates of Isaac Roberts gave negative results.

(*c*) The number of stars as bright as, or brighter than, the tenth magnitude is extremely small in all the four fields, confirming again our statements that dark nebulæ and star vacancies are correlated phenomena (see our *Miscell. Astr.*, p. 229).

(*d*) The main conclusion, however, is this, that Herschel saw exactly what we see at the Vatican Observatory, and what we call dark cosmic clouds. To Herschel belongs the credit of discovery.

Vatican Observatory :
1925 November.

Note on the Preceding Paper. By J. L. E. Dreyer.

Before acknowledging that W. Herschel was the discoverer of dark cosmic clouds, it will be well to bear in mind that he does not anywhere make any distinction between the general appearance of the objects examined by Father Hagen and the rest of the fifty-two objects.

He certainly saw, or believed that he saw, in all the fifty-two places recorded by him, luminous nebulosities. A few of them are well-known nebulæ, such as N.G.C. 7000. Considering his vast experience it is difficult to believe that he saw something totally different in the four places examined by Father Hagen, without realising this and drawing special attention to them.

On the Spiral Form of Some Nebulæ.

By Dr. H. Groot.

Through the kindness of Dr. A. van Maanen, who sent me some beautiful prints of photographs of several spiral nebulæ taken with the 60-inch reflector of the Mount Wilson Observatory, I have been able to continue my investigations as to their form.

In connection with some criticisms made by Mr. Reynolds,* it is perhaps advisable to mention that, as a matter of fact, the computed spirals from this and from my former† paper do not represent the more delicate features of the nebulæ. The spirals given show only the

* *M.N.*, **85**, No. 9, 1925 October.
† *Ibid.*, No. 6, 1925 April.

general tendency of the nebulous matter to arrange itself in curves, which to first approximation are undoubtedly equiangular spirals, but I have not tried to follow all the complicated bifurcations as Mr. Reynolds has done in his interesting investigation. Mr. Reynolds has pointed out the fact that in my former paper the nebula N.G.C. 4321 has been measured twice, once as N.G.C. 4321 and once as M. 100, without my recognising the identity of the objects. The former measurement gave respectively $\omega=27^\circ$ (23°) and $\beta=79^\circ$ (76°) for the constants of the two arms, while the latter gave $\omega=30^\circ$ (30°) and $\beta=78^\circ$ (76°). As both reproductions were on quite a different scale and did show some differences in general aspect as well, I cannot agree with Mr. Reynolds in calling this a bad agreement, which leaves a doubt as to the value of the method. I may mention further that the values which von der Pahlen * has determined for M. 51 and M. 74 agree closely with mine. So I think that both methods of treatment are still of value. Why the "coarse" structure should be an equiangular spiral, and what deviations are to be expected, are questions for further theoretical investigation.

Now, coming to the results of the present paper, it will suffice to refer to Table I., which gives a summary, and corresponds to the similar table in my former paper on this subject, with only this difference, that I have omitted the columns headed $\Delta > 1$ and $\Delta > 3$, and added the one headed "Range."

This time I did not draw the shape of the nebula on tissue-paper, but on a piece of ground glass, the rough upper surface of which was wetted to render it more transparent. I take this to be an improvement, as now the fainter and hazier parts are easily visible, which was not the case in using tissue-paper.

All nebulæ mentioned in the table turned out to be typical equiangular spirals. No other simple curves, *e.g.* Archimedian or hyperbolic spirals, can represent the drawings nearly as well over the whole extent.

The mean error ϵ comes out quite satisfactorily small in all cases, perhaps with the exception of N.G.C. 5457.

The range of β is still very limited. It seems to me that on the average the very condensed spirals have a smaller value of β than the others. In a general way this would be in agreement with the results of Mr. Reynolds.

I cannot find any different behaviour in the coarse structure between the massive spirals and the filamentous ones.

Apart from the nebulæ mentioned above, I have also studied N.G.C. 5055, N.G.C. 278, and M. 33.

N.G.C. 5055 certainly is not the ordinary spiral with two branches. It seems to me to be a sort of spiral with many arms. The extension and bad definition of the arms do not permit us to ascertain their true form.

Although hazy and badly defined, N.G.C. 278 shows three or perhaps even four spiral arms. The structure, however, is too faint to permit any trustworthy drawing being made.

* *A.N.*, 4503.

With respect to M. 33, I agree with von der Pahlen * as to the general form being an equiangular spiral if anything, but then it is so interspersed with star-like condensations, so twisted and repeatedly bifurcated, that I see no possibility of representing this object by a definite formula.

TABLE I.

Nebula.	Formula for the Spiral Arms.	ω.	β.	$\rho_{max.}$	$\rho_{min.}$	n.	Range.	ϵ.
		°	°	mm.	mm.		°	mm.
N.G.C. 4736 I.	$\log r = 1{\cdot}7324 - [8{\cdot}9100]\varphi$	33	79	54·0	9·0	17	480	2·1
II.	$\log r = 1{\cdot}7364 - [8{\cdot}8671]\varphi$	39	80	54·5	9·5	17	480	1·1
M. 74 I.	$\log r = 1{\cdot}4771 - [8{\cdot}9391]\varphi$	15	79	44·5	6·0	20	570	1·4
II.	$\log r = 1{\cdot}4843 - [9{\cdot}0519]\varphi$	15	76	34·0	6·0	17	480	0·8
N.G.C. 2403 I.	$\log r = 1{\cdot}7160 - [9{\cdot}0171]\varphi$	42	77	69·0	11·0	14	390	1·4
II.	$\log r = 1{\cdot}7160 - [8{\cdot}9815]\varphi$	39	78	57·0	12·0	12	330	1·4
G. 21 I.	$\log r = 1{\cdot}7853 - [8{\cdot}6238]\varphi$	70	85	82·0	12·0	18	510	1·3
II.	$\log r = 1{\cdot}7559 - [8{\cdot}6537]\varphi$	68	85	78·0	13·0	18	510	1·3
G. 52 I.	$\log r = 1{\cdot}5563 - [8{\cdot}8331]\varphi$	77	86	36·0	4·0	18	510	0·5
II.	$\log r = 1{\cdot}5798 - [8{\cdot}9138]\varphi$	77	86	38·0	4·0	17	480	1·8
N.G.C. 5457 I.	$\log r = 2{\cdot}0934 - [9{\cdot}2675]\varphi$	42	67	124·0	4·0	19	540	2·7

Remarks to Table I.

N.G.C. 2403 I. Structure faint in the neighbourhood of $\theta = -120°$ and $-90°$.
II. Structure rather difficult to follow.
G. 21 I. Value of ϱ_0 for $\theta = 360°$ uncertain.
II. Value of ϱ_0 for $\theta = 0$ and $360°$ uncertain.
G. 52 I. and II. Rather complex structure. The spirals here given are for the average structure and may show serious discrepancies from the finer features, but on the whole they represent the nebula fairly well.
N.G.C. 5457 I. In this object the process of condensation has gone too far to permit any rigorous representation by a single formula. The spiral here given only shows the general tendency of the matter to follow more or less an equiangular spiral, but does not represent the nebula as a whole.

Bussum :
1926 *January.*

Note on Merton's Method for the Determination of Orbits.

By R. T. Crawford.

In the June 1925 number of the *Monthly Notices of R.A.S.* Merton presents an elegant modification of Gauss's method for the determination of orbits. It is a very considerable improvement on Oppolzer's method.

The purpose of this note is to call attention to some of his statements under the heading "Comparison with other methods," which are either incorrect or misleading.

* *Loc. cit.*

Mention is made of the great advantage over the original Gaussian methods of using equatorial co-ordinates, partial elimination of parallax, and the ease with which change can be made from the parabolic to the general solution. These are indeed great advantages, but it should be added that Leuschner's method possesses exactly the same advantages as has been noted in vol. vii. *Publications of the Lick Observatory.*

Merton says: "Professor Leuschner, however, has endeavoured to make the Laplacean method practicable, and he considers that the method he has developed is superior to the others, but his claims for its shortness and advantages seem on examination to be exaggerated. It is true that the first solution is quickly obtainable, but when tested it fails to represent the observations exactly, so that in every case a differential correction is necessary, and this extra work more than offsets the shortness of the preliminary solution." In the first line of this quotation the words "endeavoured to make" should be changed to read "has made."

In part 8 of vol. vii. *Publications of the Lick Observatory*, data concerning the orbits of fourteen comets are given. On p. 388 the paragraph preceding the "Conclusion" reads: "In the first three cases (the comets for 1905) differential corrections have been used to improve the first solutions. In all other cases for short arcs the orbits represent a direct solution (no differential correction) for first orbits." Furthermore, the example given in complete detail (pp. 389–395) of the computation of the orbit of Comet 1909 *a* (Daniel) is one where no differential correction was necessary.

Of the forty-two orbits (direct solutions) computed in Berkeley by Leuschner's method since 1905, seventeen have required differential corrections. Of these seventeen, thirteen of the orbits were either based upon erroneous observations, or were ellipses of short period (5 to 8 years). Of the remaining four, two were based upon two weeks' arcs. In cases where the observations extend over two weeks a preliminary orbit based upon a shorter arc is usually available, so that a direct solution is not necessary. When a preliminary orbit is at hand Leuschner's method is that of differentially correcting this preliminary orbit to represent the observations of the longer arc. Such a process takes but little more time and labour than a direct solution without differential correction.

Our experience with Leuschner's method shows about 5 per cent. of the cases of preliminary orbits based upon an arc of a week or less require differential corrections to the direct solution. Most of these are probably necessitated by small errors of observations. It is highly probable that with perfect observations covering an arc of a week or less that a differential correction would *rarely* be necessary, instead of being necessary, as Mr. Merton states, in *every* case.

Merton says further: "Too much stress is usually laid on the fact that so much of the work can be done with fewer significant figures than in other methods." When a preliminary orbit is available and it is desired to compute a second orbit based on a longer arc, Leuschner's method, as stated, is to correct differentially the first orbit so as to represent the new observations. Using a 5-place, often a 4-place,

computation is all that is necessary to produce a 6-place orbit. This is an advantage surely worth stressing.

Professor Meyer and I have just computed the orbit of Comet 1909 *a* (Daniel) by Merton's method, and find that it took fully one-third more computing than by Leuschner's method (the same observations, 5-day arc, were used in both methods). And it may be added that every element in the orbit by Leuschner's method is closer to the best elements (Kobold's from an 8-day arc) than the corresponding elements derived by Merton's method.

Mr. Merton further says: "The fact (*Lick Observatory Publications*, **7**, 382–388) that slightly better results have frequently been obtained must not be allowed too much weight. . . ." The data of this citation show that better results have been obtained not *frequently*, but *generally*; and that *frequently* better results are obtained than in the case of orbits computed by other methods based on longer arcs (*cf.* "Conclusion," *loc. cit.*, p. 388).

In a footnote Mr. Merton says: "My first orbit computations were made by Leuschner's method, and the disadvantages only became clear to me *later* when I tried the usual forms of Gauss's and Olber's methods."

In conclusion, I wish to state that my first orbit computations were made by Olber's and Oppolzer's methods, and the disadvantages became clear to me *later* when I used Leuschner's method.

Berkeley Astronomical Department,
Berkeley, California:
1925 *October* 21.

A Reply to Professor Crawford's Note.

By G. Merton.

An opportunity has been given to me to read the above note by Professor Crawford before its publication. It is much regretted that my paper (*M.N.*, **85**, 706) contains an erroneous statement, which he rightly refutes, to the effect that in Leuschner's method a differential correction is necessary in every case. Unfortunately the paragraph referred to (*Lick Observatory Publications*, **7**, 388) escaped my notice. My experiences with the method were, I think, in every case with arcs covering a week or more, and with these I was unable to get satisfactory representations of the observations without computing differential corrections.

In the example (*loc. cit.*, p. 389) of the parabolic orbit of Comet 1909 *a* (Daniel) there remained residuals, one of which was about 8″ of arc. It must be admitted, however, that with such residuals it is permissible in the case of a diffuse comet to consider the orbit, without further correction, good enough for the immediate practical purposes. As a matter of fact, the elements thus obtained from a five-day arc were not quite as good as Kobold's from a two-day arc when compared with

the definitive orbit by Gabba,* which Professor Crawford has overlooked. Orbital elements, however, are apt to be very sensitive to even small errors in the observations, as every orbit computer knows, so that individual comparisons are not of much value and should not be stressed. That my method gave elements which were not as near the final ones as those by Leuschner's method is, therefore, of no great importance, since the *only* intrinsic test of a satisfactory result is the accuracy to which the observations are represented.†

As regards the statement that my method (for the parabola) when tried on this example required fully one-third more computing than Leuschner's method (without differential correction), it seems fair to suggest that greater familiarity with the method might have modified this result. I have admitted, however (*loc. cit.*, p. 706), that the modifications of my method, used to secure a parabolic orbit, still leave room for improvements which will shorten the process. Needless to say, where a preliminary orbit is already available, the solution for a fresh orbit is much shortened, and certain higher order quantities can be included at once.

With regard to the elimination of parallax, it should be pointed out that in my method the elimination is complete, and not partial, as Professor Crawford implies. In Leuschner's method a complete elimination involves considerable complications (*loc. cit.*, pp. 313, etc.), but the partial elimination he nearly always uses seems generally to be sufficient.‡

My remark to the effect that too much stress is usually laid on the fact that so much of the work of computing an orbit by Leuschner's method can be done with fewer significant figures than in other methods, was made because there are points for consideration which appear to be of more practical importance than this, such as the total amount of work involved, the means of computation (calculating machine or logarithms), the number and nature of the different quantities and formulæ to be handled, and, especially for the single-handed computer, the numerical checks available. Circumstances may modify these considerations, as, for example, where two computers work together, as appears generally to be the practice at Berkeley. At any rate, it is suggested that these practical aspects merit further study.

* R. Oss. Brera, 52 and 57; also given in Crommelin's *Comet Catalogue* (1925).

† A comparison with the definitive orbit by Gabba indicates that the errors of these observations (Observed-Definitive), cos $\delta.d\alpha$, and $d\delta$, are: $-3''$, $-1''$; $-8''$, $-12''$; $0''$, $+1''$. Errors, like those in the middle place, must considerably affect the elements of an orbit deduced from such a short arc.

‡ The example 1900 GA, mentioned in my paper (*loc. cit.*, pp. 707–708) in another connection, is unfortunately an old one, and it should be noted that Leuschner subsequently introduced the improvements for dealing with the parallax.

The Royal Observatory, Greenwich:
1925 *Dec.*

The Personality of Mr. C. Grover and the Reduction of the Rousdon Observations. By P. M. Ryves.

1. In the Supplementary Number, *M.N.*, **85**, 9, Professor H. H. Turner draws attention to certain peculiarities in the personality of Mr. Grover in estimating light intervals, which peculiarities he has detected as a result of an examination of a considerable portion of the Rousdon observations.

A similar result had been obtained previously (*Mem. R.A.S.*, **55**, pp. x–xvii), but in the later discussion fresh material has been used and some refinements introduced.

The most important point of Professor Turner's investigations is his finding that "Mr. Grover's estimations of magnitude differences . . . do not have a constant ratio to the actual differences except (presumably) for a small range of difference between +0·4 and −0·4. Outside these limits they numerically exceed the actual differences by a sensibly constant amount." Professor Turner adds: "This constancy of the error for intervals exceeding half a magnitude was the most striking feature of Mr. Grover's personality, and it seems to have remained sensibly unaltered in his later years"; finally arriving at the conclusion that the formula (*c*) given at the foot of p. xvi of the Memoir cited is applicable to the later observations.

2. Referring to the last vertical column of "Table I." in Professor Turner's paper: If we take the arithmetical means of the values corresponding to +0·1 and −0·1, +0·2 and −0·2, and so on, we obtain the following figures for the mean corrections to the sixteen estimated intervals from 0·1 to 2·0: ·04, ·055, ·085, ·08, ·105, ·13, ·185, ·175, ·12, ·20, ·22, ·17, ·18, ·205, ·25, and ·24. These are shown as dots in the accompanying figure, plotted against the estimated intervals.

Now it is obvious from this diagram that, in estimating differences up to about 1·0 mag., the mean value of Mr. Grover's grade was sensibly constant and about equivalent to 0·08 mag.; whilst, when larger intervals were involved, considerable irregularities are apparent and the value of the grade seems to increase. (Although the estimates are recorded as magnitudes, I prefer to consider each estimated tenth of a magnitude as 1 grade.)

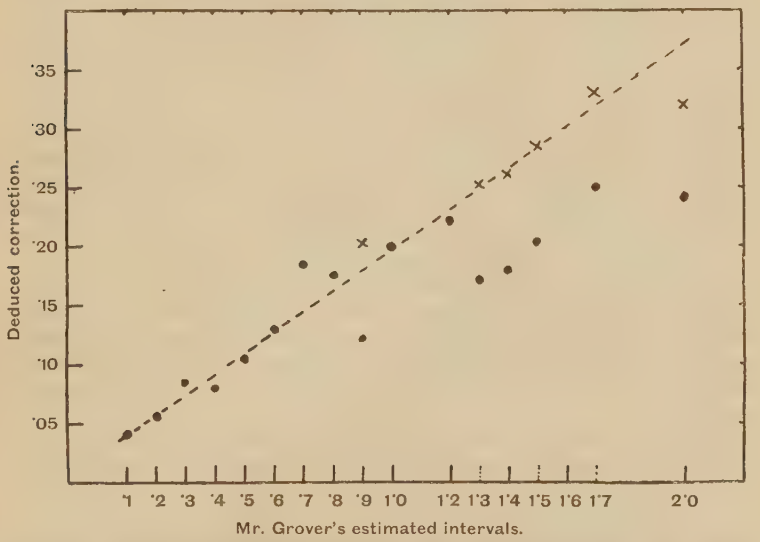
There is nothing particularly remarkable about this result. The constancy of the grade value up to the rather large difference of 12 grades is very satisfactory, whilst the wider estimates show certain irregularities, as might be expected where we have to deal with what can be little more than good guesses. The "correction" to the 9-grade group is discordant; but more of this later.

We may say, then, that whenever the differences estimated were of a reasonable size, the value of Mr. Grover's grade is the same for smaller and for larger intervals, and that anything abnormal that there may be about his personality refers principally to the very difficult estimates of comparatively smaller worth. From the point of view of the reduction of the observations the result is satisfactory, and probably most

editors would elect to use the moderate estimates and a rectilinear formula of reduction, discarding the wider ones.

3. But we have a great respect for Mr. Grover's observations ; moreover, Professor Turner has found certain peculiarities in his personality which appear sufficiently extraordinary to require explanation.

Here it may be remarked, however, that the fact that outside certain limits " the estimates numerically exceed the actual differences by a sensibly constant amount " might be explained by supposing that beyond a certain light difference the grade value increases, and that the rate of increase happens to be such that the correction, or adjustment to star magnitude, remains nearly constant. It will be noticed that the point where the increase begins is just about at the



separation between reasonably small and inconveniently large differences. In other words, whenever it was a case of estimating reasonable light intervals, the observer's grade was constant; but beyond that limit the grade becomes larger and larger with increase of light interval.

Such an increase in the grade value might be attributed to a natural conservatism or caution on the part of the observer when estimating large intervals. After having received a first impression, which was perhaps correct, he, as it were, says to himself, " it surely can't be as much as that," and records a smaller value. There are people who habitually fall short in their estimates of large quantities though they may judge correctly of smaller ones. This over-impressionability or lack of imagination in the presence of large quantities, leading to a curtailed perspective or dropping in the curve of estimation, is not uncommon, especially near the end of a scale, and may be traced in the magnitude estimations of the Bonner Durchmusterung. It tends to

become obtrusive when a scale or method is pushed beyond its proper limits.

4. But an examination of the data contained in Professor Turner's table suggests that there are other influences to be considered.

It will be noticed, firstly, that after reaching the estimated interval of 1·0, or 10 grades, Mr. Grover's scale becomes more closed up, or coarser. He jumps from 1·0 to 1·2, from 1·7 to 2·0, and, I think also, from 2·0 to 2·5 (though this is beyond the limit of the table); *but he records the three consecutive groups* 1·3, 1·4, *and* 1·5. Secondly, the number of estimates in group 1·4 is small; and thirdly, the number of estimates in group ·9 is small, and the value of mean correction also small and discordant, the dot on the diagram falling well below the curve.

It is clear that on reaching 10 grades, nominally 1·0 mag., the observer began to estimate in double grades; indeed, he had already begun to do so after passing 0·8, as is shown by the fact that there are only 76 estimates recorded as 0·9 against 520 in group 0·8 and 1703 in group 1·0.

Estimating in double grades, after 1·0 the next larger difference observed was 1·2, and after 1·2 the next interval *estimated* was 1·4, but this latter was erroneously *entered* as 1·3. In the same way the following estimate 1·6 was entered as 1·5; and having got so far, we might perhaps be justified in supposing that the remaining groups 1·7 and 2·0 would require moving on by 0·1 in order to bring them in line with the previous groups, should we decide to correct the error of entry.

Now, if this is a proper proceeding, and the smallness of the number of estimates in group 1·4 supports this view, it follows that the "corrections," or adjustments to star mag., corresponding to these groups must be increased by a quantity not exceeding 0·10 and the dots on the diagram raised not more than 0·08, as shown by the position of the crosses.

If this is done the anomaly practically disappears. There is now no constant correction; on the contrary, the grade is constant right up to 1·7, and nearly so at 2·0. The rather small value of the correction to the latter group might be explained by assuming a caution or contraction of scale, such as has been already alluded to, or it might arise through an error in the new group adjustment. When dealing with large differences like 2 magnitudes, and when an observer groups his estimates under headings such as 1·7, 2·0, and 2·5, etc., a small error in the designation of a group or fixing of its "centre of gravity" may easily cause a discrepancy of a few hundredths in the value of the corresponding deduced mean correction.

That the above is a correct solution, we have further and independent evidence in the smallness of the value for the correction to the group 0·9. On our theory these estimates, or most of them, should have been entered as 1·0, which would make the correction larger by some quantity not exceeding 0·1. The fact that such a correction would remove the discordance is evidence of the correctness of the supposition.

5. But it may be asked, How could a careful observer like Mr. Grover make these mistakes of entry?

The answer may be found in a consideration of personal scale or decimal equation.

To take a familiar example : It is well known that in estimating, to a tenth of a second of time, the moment at which a star passes behind a wire of the transit instrument, an observer will generally show a preference for certain numbers whilst avoiding others, although all should occur with equal frequency. The surplus estimates assigned to a favoured number are drawn from the neighbouring groups, not necessarily in equal proportions.

That Mr. Grover had a decided preference for the number 3 is evident from the fact that in "Table I." there are 845 estimates of 0·3 against 765 of 0·2, and only 297 of 0·4. He also shows a preference for that general favourite, number 5.

We may now reconstruct the process as follows :—

Up to 8 grades (0·8 mag. nominal) Grover estimated light intervals in normal or single grades of about 0·08 mag. After ·8 he began to use double grades, so that the next larger estimate was 10 and was recorded 1·0 (except on a few occasions, when he apparently forgot and entered 0·9), and after 10 the following estimate was 12, which was always entered correctly, the group 1·1 being entirely absent. Continuing, the next larger estimate made was 14, but owing to the strong attraction which the number 3 possessed for the observer, this was generally entered as 1·3. In a similar manner 16 was entered as 1·5, partly on account of the attraction of the number 5, and also, perhaps, because the previous smaller estimate had been recorded as 1·3 instead of 1·4.

One detail may be mentioned here. On passing from single to double grades the transition might not be absolutely sudden. It is possible, therefore, that some of the 76 estimates entered as 0·9, and which we have assumed should have been entered as 1·0, were really estimates of 9 single grades. This would mean that the point indicated by the position of the cross on the diagram would represent an over correction, the true point being somewhat lower, and thus probably in still better agreement with the other observations.

6. In order to interpret the Rousdon observations, then, we must first make the adjustments mentioned above, and which, broadly speaking, consist in adding 0·1 mag. to the intervals estimated as more than 1·2, and a similar quantity to those recorded as 0·9.

7. Professor Turner refers briefly to the effect of magnitude on the estimates, stating that the mean value of the correction must be increased when the stars compared are bright, and diminished when they are faint.

In view of the presence of magnitude equation, it is unfortunate that in compiling "Table I." the argument "mean mag. of pair" was not taken instead of "mag. of comp. star," which renders the disentanglement of the effect somewhat difficult. In order to partially remedy this defect, the mean corrections for the sixteen estimated intervals have been rededuced, combining the values for "correction" in the first (vertical) column of the upper half of the table with those in the second column of the lower half, and so on for the other columns. In this way four groups of mean values are obtained. In addition, the corrections in the first lower half column (comp. star $> 8{\cdot}5$, var. $>$ comp. star) and those in the last upper half column (comp. star $< 10{\cdot}1$,

var. <comp. star) form two outside groups, the former being the brightest and the latter the faintest.

The estimates in each group are further divided into three sections —small, medium, and large differences—comprising intervals of 0·1 to 0·5 inclusive; 0·6 to 1·2, omitting 0·9; and 1·3 to 2·0, corrected for personal scale. The means of these sections, converted into grades, are shown in the table.

Mean Value of Grade expressed in Star Magnitude.

	Light Differences.			
	Large. ·1–·5.	Medium. ·6–1·2.	Small. 1·3–2·0.	All. ·1–2·0.
Group 1, faintest	·084	·086	·091	·087
„ 2	·088	·086	·089	·088
„ 3	·087	·083	·063	·078
„ 4	·089	·076	·070	·078
„ 5	·080	·063	·058	·067
„ 6, brightest	·076	·063	·033	·057
Mean	·084	·076	·067	·076

It will be seen that in all cases there is a decrease in the size of the grade with increase in the brightness of the stars compared, but *this decrease is greater when the estimated differences are small than it is when they are large*, becoming extreme in the brightest groups.

As far as the reduction of the observations is concerned, when the light intervals are large magnitude equation of the grade could be ignored without introducing a very large error; and the same might be done in the case of moderate differences, except in the brightest or two brightest groups. But with the small differences the error becomes serious; it can be seen directly in "Table I.," where the corrections to +0·2, +0·3, and +0·4 are of the order of 100 per cent.

This curious result may be explained in the following way :—

When the variable is bright, say 7th mag., and is observed with a 6-inch telescope, it must frequently occur that there will be no other stars of similar brightness within a convenient apparent angular distance, but there will nearly always be some stars of 9th or 10th mag. within such limits, say in an area of one square degree. Thus, in an observation consisting of several comparisons, those which involve the estimation of large differences will generally refer to stars situated near the variable, there being no need to seek them further away; whereas most of the comparisons involving the estimation of small differences (bright comparison stars) will be with stars situated at a greater apparent angular distance and visible only near the edge of the field of view, or even outside altogether. In one case the difficulty consists in the large light difference, in the other it is the large apparent angular separation; but the observer will avoid the concurrence of both evils in any one comparison, and this would be generally possible even in barren regions of the sky.

It follows that the estimates involving large light differences would, in general, be simultaneous comparisons, whilst many of those involving small light differences would be successive comparisons, the observer looking alternately from one star to the other. Now, successive comparisons are on quite a different footing from simultaneous comparisons, a new factor (memory) is introduced, and it is possible that the grade value would be quite different.

The point could be decided by reference to the original observations, when it would probably be found that when the variable was compared with stars at apparent angular distances of more than 25°, the "correction" would be considerably greater than when the apparent angular separation was small, irrespective of the magnitude of the stars compared.

8. There are always disadvantages in pushing a method beyond its proper limits, and this is what had to be done at Rousdon in order to comply with the conditions of an inflexible plan. Most observers who adopt the method of Argelander, when they cannot command more than one or two convenient comparison stars, content themselves with one or two comparisons. Mr. Grover was, in a sense, under the obligation of making five comparisons at every observation, and he fulfilled this obligation, firstly, by making estimates of very large light differences, and secondly, we may assume, by seeking comparison stars at large apparent distances from the variable. The former procedure must have involved large accidental errors, though, after certain adjustments are made, the systematic errors appear to have been remarkably small; the latter probably entailed a change in the method of estimation, necessitating the use of a special grade value for these particular comparisons.

9. In the concluding paragraph of his paper Professor Turner remarks that Mr. Grover's estimates of large intervals are astonishingly accurate in the mean number "though they must be very rough in the individual cases"; but he does not give any information as to what the probable errors actually are. Yet it is difficult to see how the observations can be reduced to star magnitude without some knowledge of the relative reliability of the estimates of various differences. When, in the reduction of an observation, we have to combine estimates of intervals as large as 2 or 3 magnitudes with others of a few tenths, we obviously cannot give them all the same weight. Some method of weighting must be adopted, and in order to be able to devise a suitable system it is necessary to have some idea of the relative probable errors or mean divergences of the estimates of large and small differences.

In conclusion, I should like to point out that, in publishing a fine series of observations, such as those of Rousdon, the really important thing is not the reduction but the printing of the individual comparisons, together with as much information as possible as to the circumstances under which they were made. As long as this is done, subsequent investigators can effect the reduction, each one in accordance with the information and experience which he possesses, and in a manner suitable to the particular end for which the observations are required.

Summary.

1. Mr. Grover's estimates of differences of magnitude (light intervals) are subject to certain personal scale effects ; if these are allowed for, the mean value of his grade is remarkably uniform over a wide range of magnitude differences.

2. As stated by Professor Turner, the estimates are subject to magnitude equation, but it is considerable only when the light differences are small and the stars compared bright.

3. In order to reduce the Rousdon observations it is desirable to know the mean divergences or probable errors corresponding to estimates of different light intervals.

4. After making adjustments for personal scale effects and duly weighting the estimates, the bulk of the Rousdon observations might be reduced in a very simple manner, assuming the grade value to be constant.

5. If certain conclusions arrived at in the present investigation are correct, a portion of the estimates, corresponding to the extreme case —stars bright, difference small—require reduction by a special method.

6. It is particularly important that the observations should be published in detail.

Zaragoza, Spain :
1926 *January* 5.